Numerical Modelling of Poroviscoelastic Grounds in the Time Domain Using a Parallel Approach

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Abstract. In this paper, we present a parallelized finite element code developed to study wave propagation phenomena, specifically in porous soils problems which usually require millions of degrees of freedom. The parallelization technique uses an algebraic grid partitioning managed by a Single Program Multiple Data (SPMD) programming model. Message Passing Interface (MPI) library specification is the standard used to exchange data between processors. The architecture of the code is explained and numerical results show its performance.

1 Introduction

The study of the mechanical wave propagation in porous media is a subject of great interest in diverse scientific fields ranging from environmental engineering or vibration isolation to geomechanics. At the macroscopic scale, the medium is considered as a two-phase continuum. The Biot theory is known as the reference theory to deal with the macroscopic mechanical wave propagation phenomenon, see Biot [1] or Coussy [2] for instance.

Theoretical works are restricted to simple geometries. Consequently, they have to be completed by numerical approaches such as Finite Element or Boundary Element Methods, allowing the study of more complex problems to better represent the ground. The difficult study of transient regimes in geomechanics has been treated numerically by several authors but only for specific cases, Zienkiewicz and Shiomi [3], Simon et al. [4] and Gajo et al. [5] for example. In particular, in many cases, the tortuosity and the physical damping parameters are not taken into account.

Moreover, even with an efficient and optimized finite element code, only a restricted range of problems can be treated. As a matter of fact, solution of practical problems (for instance, realistic 3D geometries, and problems with short pulse load needing fine meshes for representing well the high frequencies)

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usually requires millions of degrees of freedom. This is often virtually out of capabilities of contemporary sequential computers either because of lack of memory or abundantly long computation time. In all these cases, parallel programming techniques may be a good solution to overcome the computational complexity.

Nevertheless, to our knowledge, no parallelization of numerical approach on the complete Biot theory in the transient domain exists in the literature. In fact, papers presenting parallel computing of the finite element method often deal with elastodynamic problems using the domain decomposition method, Papadrakakis and Bitzarakis [6], Lirkov [7] and Bohlen [8].

In this paper, the authors propose a parallelized version of a finite element C++ code specifically developed at the CSE Laboratory to study transient wave propagation. This approach includes the whole Biot theory with all the couplings which represent the interactions between the solid and fluid phases. The sequential version has previously been presented at ICCS 2005, Mesgouez et al. [9].

The study of the different time-consuming parts yields a parallelization technique using an algebraic grid partitioning managed by a SPMD programming technique. MPI standard library is used to exchange data between processors. For this, numerical results, obtained for a two-dimensional problem, include the analysis of speed-up and efficiency on a SGI Origin 3800. Complementary results compare the performance obtained by another supercomputer (AMD-ATHLON cluster), on which MPICH and SCI-MPI implementations of MPI standard is used. First results with a three-dimensional geometry solving a problem with more than 2,000,000 unknowns are obtained.

2 Mechanical and Numerical Works

2.1 Spatial Scales and Macroscopic Approach

When we focus our attention on the description of a porous medium, the first question to be put is that of the spatial scale of analysis: indeed, two approaches are conceivable. The first one is situated at the microscopic scale. The characteristic length size is the dimension of the pore. In this configuration, the solid matrix is partially or completely filled with one or several viscous fluids. One geometric point is thus located in one of the different identifiable solid or fluid phases. Mechanical equations of each phase and mixture with compatible interface conditions are written. They correspond to those of linear elasticity in the solid and those of Stokes in the fluid. This approach deals with problems like interface modelling or description of microscopic geological structures. Homogenization is then obtained through asymptotic developments or averaging procedures and leads to a macroscopic description of the porous medium, see Terada and al. [10] or Coussy et al. [11] for instance. We obtain thus the famous set of macroscopic mechanical equations for a representative elementary volume. In this macroscopic spatial description, the porous medium is seen as a two-phase continuum. This scale, we study here, is well adapted to most of practical geomechanical problems.

Writing u_i and U_i respectively the macroscopic solid and fluid displacements components, Biot's equations can be written with usual notations as follows:

$$\sigma_{ij,j} = (1-\phi)\rho_s \ddot{u}_i + \phi\rho_f \ddot{U}_i \tag{1}$$

$$p_{,i} = -\frac{\phi}{K}(\dot{U}_i - \dot{u}_i) + \rho_f(a-1)\ddot{u}_i - a\rho_f \ddot{U}_i \tag{2}$$

$$\sigma_{ij} = \lambda_{0v}\varepsilon_{kk}\delta_{ij} + 2\mu_v\varepsilon_{ij} - \beta p\delta_{ij} \tag{3}$$

$$-\phi\,(U_{k,k} - u_{k,k}) = \beta u_{k,k} + \tfrac{1}{M}p \tag{4}$$

σ_{ij} are the total Cauchy stress tensor components and p is the pore pressure. The soil's characteristics are: λ_{0v} and μ_v (drained viscoelastic equivalent Lamé constants), ρ_s and ρ_f (solid grains and fluid densities), ϕ (porosity), K (hydraulic permeability representing the viscous coupling), a (tortuosity standing for the mass coupling), M and β (Biot coefficients including the elastic coupling).

2.2 Finite Element Formulation and Numerical Resolution

To determine the solid and fluid displacements in the ground, we develop a numerical code based on the finite element method for the space integration, coupled to a finite difference method for the time integration. The main steps are:

- some boundary and initial conditions are associated to the previous partial differential system. Some modifications on the field equations are done in order to lead to a Cauchy's problem.
- integral forms are obtained using the weighted residual method. They are then spatially and analytically discretized and lead to a time differential system. The global differential system to be solved can be written as

$$[M]\frac{d}{dt}\{W^{(G)}\} + [K]\{W^{(G)}\} = \{F^{(G)}\} \tag{5}$$

  $[M]$ and $[K]$ are respectively the global mass and stiffness matrixes. $\{W^{(G)}\}$ and $\{F^{(G)}\}$ are the global vectors of unknowns and solicitation. With the developed technique, the mass matrix is diagonal and can be easily inverted.
- the backward finite difference method modified with an upward time parameter is used to obtain an approximate solution of the problem.

2.3 Structure of the Code and Parallelization

The sequential code called FAFEMO (Fast Adaptive Finite Element Modular Object), developed to solve the previous problem, constitutes an efficient code to deal with transient 2D problems and small 3D ones. The use of a matrix free technique, not necessary for small cases, becomes interesting for huge ones. An expert multigrid system is also used to optimize the problem size and yields a modification of the global matrixes at each time step. The two previous techniques lead to a high performance level both for the storage and the CPU costs.

Table 1. Time profile of the 2D sequential code

reading of the data files and *element* class	*elementary matrixes* class	*building-resolution* class
7.45%	90.60%	1.95%

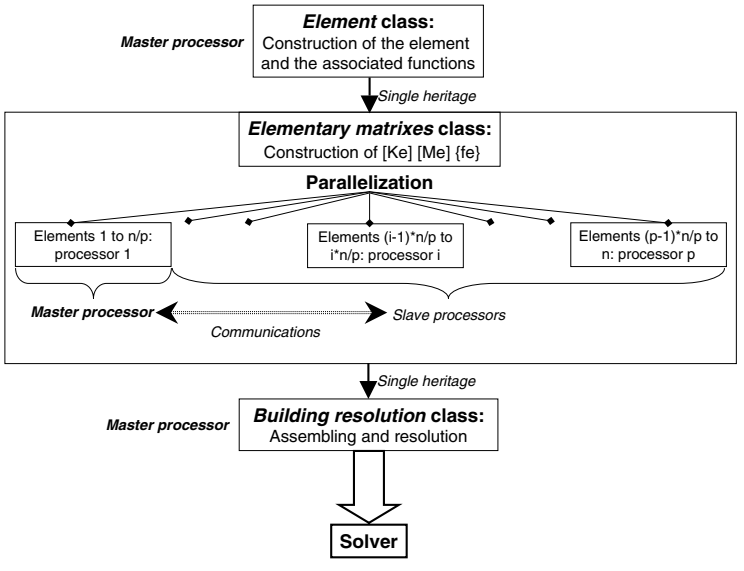

Fig. 1. Structure of the parallelized version of the C++ finite element code

The C++ code is organized in three classes connected by a single heritage: *element*, *elementary matrixes* and *building-resolution* classes.

For huge problems, the elementary vectors have to be calculated and assembled for each time step since they are too expensive in terms of Input/Output cost to be stored. In order to treat 3D problems and to perform intensive 2D parametric studies, we propose a parallelization of the code to reduce the time calculation.

The Unix/Linux *gprof* tool draws a time profile of the sequential code. For a two-dimensional application, the elapsed time is divided as presented in Table 1, for each of the three classes.

The part which is the largest consumer of elapsed time clearly appears to be the *elementary matrixes* class. This can be explained as the elementary matrixes have to be calculated for each time step. Besides, as we use a matrix free technique with a diagonal mass matrix, the resolution part is more efficient and needs little computational time. Moreover, the process of construction of $[K_e]$, $[M_e]$ and $\{f_e\}$ is iterative and independent element by element. This independent and time-consuming loop can thus be divided into several processors by distributing the n elements between p equitable parts.

We use a grid partitioning based on algebraic decomposition which is performed randomly without any geometric factors. Several advantages are:

- unlike the domain decomposition method, this technique does not need any particular interface management. This is particularly important when the expert multigrid system is activated, or when the geometry is changed.
- moreover, when the size of the grid is modified, the algebraic distribution of the elements leads to an equitable load balancing between processors at each time step.
- another advantage of this approach is that the implementation is as close to the original sequential solver as possible.

A SPMD programming model manages the algebraic distribution of the different finite elements of the grid and MPI standard library is used to exchange data concerning the elementary matrixes between master and slave processors. The architecture of the parallelized version of the code is summarized on figure 1.

3 Results

In this section, numerical results are presented for a 2D problem involving 200,000 unknowns to estimate the performance of the parallelized version of the FAFEMO code.

3.1 SGI Origin 3800

We have worked on SGI Origin 3800 installed at the National Computer Center of Higher Education (CINES, Montpellier, France). SGI ORIGIN is a shared global memory based on cc-NUMA architecture (cache-coherent Non-Uniform Access Memory) composed of 768 processors MIPS R14000 (500 MHz, 512 Mo RAM), distributed among 80 nodes. The internal network (Bristled hypercube) give a speed of 1,6 GB/s.

Figures 2 and 3 present the evolution of the elapsed times and the speed-up depending on the number of processors (up to 64). The main conclusions are:

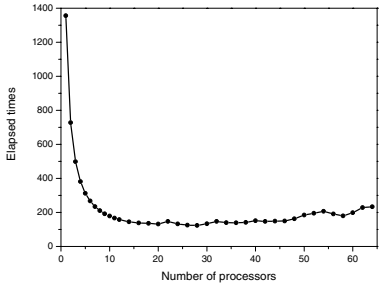

Fig. 2. Elapsed times in minutes up to 64 processors on SGI Origin 3800

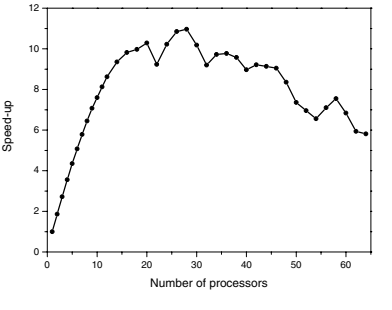

Fig. 3. Speed-up up to 64 processors on SGI Origin 3800

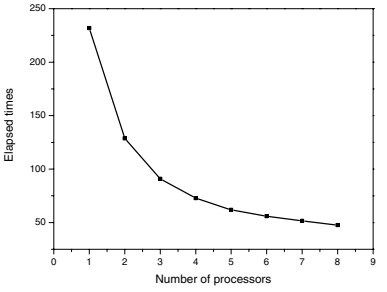

Fig. 4. Elapsed times in minutes up to 8 processors on cluster CHOEUR

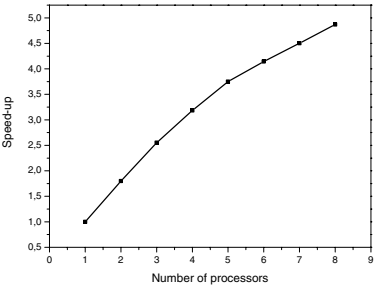

Fig. 5. Speed-up up to 8 processors on cluster CHOEUR

- the elapsed time is reduced considerably for a few number of processors and the communications do not affect the performance of the parallelization. For instance for 8 processors, the speed-up value equals 6.45. Compared to literature results on elastodynamic problems, these results are pretty good.
- for a larger number of processors, the performance of the code is not so interesting. Actually, the maximum of the speed-up curve corresponds to an optimal value of the number of processors situated between 20 and 30.

3.2 Cluster and Comparison

We have also worked on cluster CHOEUR, installed at the Laboratoire de Modélisation et Simulation Numérique en Mécanique et Génie des Procédés (MSNM-GP/L3M) of the CNRS and Aix-Marseille University. It is an AMD-ATHLON cluster, built upon 20 AMD-760MP 1800 bi-processors nodes interconnected with gigabit Ethernet, and 6 AMD-760MPX 1800+ bi-processors nodes (1530 Mhz, 1 GB RAM) interconnected with SCI-Dolphin High Speed 2,5 Gb network. The cluster is managed by an extra master node (2KMhz, 2 GB RAM).

Figures 4 and 5 present the elapsed times and the speed-up. Results are limited to 8 processors. As previously, the results show a real gain for the time calculation. Nevertheless, we can not visualize the maximum of speed-up for cluster CHOEUR which does not provide enough processors to reach it.

The elapsed times on SGI Origin 3800 are much longer than the ones obtained on cluster CHOEUR: for a single processor, more than 1300 min (i.e. 22 h) on SGI Origin 3800 and only 230 min (i.e. less than 4 h) on CHOEUR, because processors on CHOEUR are more recent and efficient.

The speed-up and the efficiency are better on SGI than on CHOEUR: for instance for 5 processors, the speed-up obtained on SGI equals 4.3 and 3.7 on CHOEUR. Calculation time is longer dominating compared to communications time on SGI due to less efficient processors.

Moreover, the inter-processor communication process is a very important parameter to take into account. Particularly, due to the hierarchical architecture of the memory of SGI (NUMA, non uniform memory access), the communication times decrease with the number of processors, but only over a limited range of

processors. Concerning CHOEUR, communications are optimal since the limits of the mother board PCI bus are not reached.

Consequently, the difference in performance between SGI and CHOEUR is mainly due to the difference of quality of the two kinds of processors and the management of communications.

3.3 Physical Example

Previous 2D results, obtained with the sequential version of the code, have been presented for instance in [9] and [12]. Parametric studies on the influence of the different couplings and an approach of heterogeneous soil have been carried out.

The interest of the parallelized version is to be able to lead parametric study more rapidly and to tackle problem much more expensive in terms of storage and CPU costs, like 3D geometries. The 3D problem presented here is a 2,300,000 unknowns problem and concerns an homogeneous poroviscoelastic semi-infinite half-space ground subjected to an impulsional load applied on the center of the surface. Figure 6 presents for instance the solid displacement contour levels for a given time on the surface of the ground. The propagation of the Rayleigh wave is clearly perceptible and corresponds to the dark area. The first compressional wave has almost reached the boundary. Figure 7 shows similar results for the fluid phase for a high hydraulic permeability value: note that the two behaviors are strongly uncoupled. These results allow great perspectives for the study of 3D heterogeneous soils.

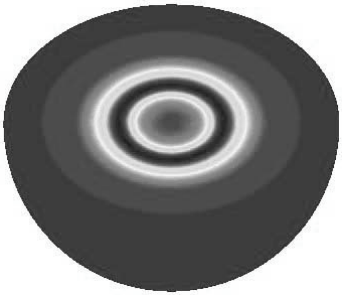

Fig. 6. Contour levels of the solid displacements (dimensionless time = 0.6)

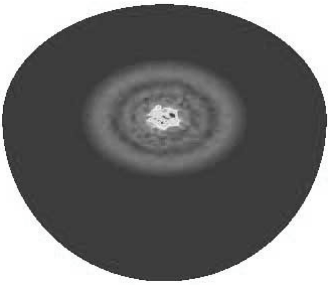

Fig. 7. Contour levels of the fluid displacements (dimensionless time = 0.6)

4 Conclusion and Further Works

A parallelized finite element code has been presented to study wave propagation phenomena in poroviscoelastic grounds. In fact, the applications are wider and can concern for instance porous bones or foams. Besides, the code can treat all propagation wave phenomena: a version studying electromagnetic wave propagation have been developed in the same way.

With the parallelized code, one can consider the work to be continued: *i)* a study of more complex ground : an approach of partially saturated ground is under progress *ii)* an analysis of 2D heterogeneous random media mixing two kinds of soils has shown the existence of different thresholds for fluid and solid phases: we would like to do similar analysis for a 3D geometry in order to know if conclusions are identical or not *iii)* a study of the coupling of electromagnetic and seismic waves, which represents a hard problem because of the two different time scales.

Moreover, concerning the numerical point of view, some improvements can still be done: for instance a compressed message passing technique for inter-node communication could allow a significant reduction of the communications time.

To achieve these aims, an access of more computational resources at the CINES has been asked for 2006. Besides, SGI will be soon replaced by a massive parallel supercomputer which will allow more efficient abilities.

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