Data Dependent Wavelet Filtering for Lossless Image Compression

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Abstract. A data dependent wavelet transform based on the modified lifting scheme is presented. The algorithm is based on the wavelet filters derived from a generalized lifting scheme. The proposed framework for the lifting scheme permits to obtain easily different wavelet FIR filter coefficients in the case of the (~N, N) lifting. To improve the performance of the lifting filters the presented technique additionally realizes IIR filtering by means of the feedback to the already calculated wavelet coefficients. The perfect image restoration in this case is obtained employing the particular features of the lifting scheme. Changing wavelet FIR filter order and/or FIR and IIR coefficients, one can obtain the filter frequency response that match better to the image data than the standard lifting filters, resulting in higher data compression rate. The designed algorithm was tested on different images. The obtained simulation results show that the proposed method performs better in data compression for various images in comparison to the standard technique resulting in significant savings in compressed data length.

Keywords: image processing, wavelets, lifting scheme, adaptive compression

1 Introduction

In the past decade, the wavelet transform has become a popular, powerful tool for different image and signal processing applications such as noise cancellation, data compression, feature detection, etc. Meanwhile, the aspect of wavelet decomposition/reconstruction implementation, especially for image compression applications, now continues to be under consideration.

The first algorithm of the fast discrete wavelet transform (DWT) was proposed by S.G.Mallat [1]. This algorithm is based on the fundamental work of Vetterli [2] on signal/image subband decomposition by 1-D quadrature-mirror filters (QMF), and orthonormal wavelet bases proposed by I.Daubechies [3]. Then, W.Sweldens [4] proposed the lifting scheme based on polyphase factorization of known wavelets that now is widely used (for example, in JPEG2000 standard) for lossless image/signal compression based on DWT. To enhance the energy compaction characteristics of the DWT, different methods basing on an adaptive lifting scheme [4 - 8], principal

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components filter banks [9] and signal-dependent wavelet/subband filter banks [10-12] were developed recently.

In this paper, we present an algorithm for lossless image compression that is based on a subclass IIR wavelet filters. These filters are derived from the generalized FIR wavelet lifting filters [8, 13] introducing poles in the prototype FIR filters. Performing causal filtering at the analysis and anticausal filtering of the time-inverted data at the synthesis stages, one can obtain the perfect image restoration with the presented IIR filters. Varying the order of the filter and the filter coefficients depending on the image data statistical/spectral properties, the decompositions can be optimized to achieve a minimum of the entropy in the wavelet domain.

2 Generalization of the Lifting Scheme

The lifting scheme [3] is widely used in the wavelet based image analysis. Its main advantages are: the reduced number of calculations; less memory requirements; the possibility of the operation with integer numbers. The lifting scheme consists of the following basic operations: splitting, prediction and update.

<u>Splitting</u> is sometimes referred to as the lazy wavelet. This operation splits the original signal $\{x\}$ into odd and even samples:

$$s_i = x_{2i}, \quad d_i = x_{2i+1}. \tag{1}$$

<u>Prediction,</u> or the dual lifting. This operation at the level k calculates the wavelet coefficients, or the details $\{d^{(k)}\}$ as the error in predicting $\{d^{(k-1)}\}$ from $\{s^{(k-1)}\}$:

$$d_i^{(k)} = d_i^{(k-1)} + \sum_{j=-\tilde{N}}^{\tilde{N}} p_j \cdot s_{i+j}^{(k-1)}, \tag{2}$$

where $\{p\}$ are coefficients of the wavelet-based high-pass FIR filter and $\tilde{N}$ is the prediction filter order.

<u>Update</u>, or the primal lifting. This operation combines $\{s^{(k-1)}\}$ and $\{d^{(k)}\}$, and consists of low-pass FIR filtering to obtain a coarse approximation of the original signal $\{x\}$:

$$s_i^{(k)} = s_i^{(k-1)} + \sum_{j=-N}^{N} u_j \cdot d_{i+j}^{(k)}, \tag{3}$$

where $\{u\}$ are coefficients of the wavelet-based low-pass FIR filter and N is the prediction filter order.

The inverse transform is straightforward: first, the signs of FIR filter coefficients $\{u\}$ and $\{p\}$ are switched; the inverse update followed by inverse prediction is calculated. Finally, the odd and even data samples are merged.

A fresh look at the lifting scheme first was done in [13], where the FIR filters that participate in the prediction and update operation are described in the domain of Z-transform. According to this approach, the transfer function of the prediction FIR filter can be formulated as follows [8]:

$$H_p(z)=1+p_0\left(z+z^{-1}\right)+p_1\left(z^3+z^{-3}\right)+\ldots+p_{\tilde{N}-1}\left(z^{2\tilde{N}-1}+z^{-2\tilde{N}+1}\right), \tag{4}$$

The $H_p(z)$ must has zero at $\omega=0$, i.e., at $z=1$. It can be easily found [5] that this condition is satisfied when

$$\sum_{i=0}^{\tilde{N}-1} p_i=-\frac{1}{2}. \tag{5}$$

When the condition (5) is satisfied, $H_p(-1)=2$ and $H_p(0)=1$ that means the prediction filter has gain 2 at $\omega=\pi$ and unit gain at $\omega=\frac{\pi}{2}$.

Following this approach, the transfer function for update filter can be obtained in the terms of $H_p(z)$ [8]:

$$H_u(z)=1+H_p(z)\left\{u_0\left(z+z^{-1}\right)+u_1\left(z^3+z^{-3}\right)+..+u_{N-1}\left(z^{2N-1}+z^{-2N+1}\right)\right\}. \tag{6}$$

Similarly, $H_u(z)$ must has zero at $\omega=\pi$, i.e., at $z=-1$. It can be easily found [8, 13] that this condition is satisfied when

$$\sum_{i=0}^{N-1} u_i=\frac{1}{4}. \tag{7}$$

When the condition (7) is satisfied, $H_u(1)=1$ and that means the prediction filter has gain 1 at $\omega=0$.

An elegant conversion of the formulas (5), (7) in the case of (4,4) lifting scheme was proposed in [13] to reduce the degree of freedom in the predictor and update coefficients. With some modifications, the formulas for the wavelet filters coefficients are as follows:

$$p_0=-\frac{128+b_p}{256},\quad p_1=\frac{b_p}{256}, u_0=\frac{64-b_u}{256}, u_1=\frac{b_u}{256}, \tag{8}$$

where b_p and b_u are the parameters that control the DWT properties. The correspondences between these control parameters and the conventional (non-lifted) biorthogonal wavelet filters can be found in reference [13].

Using the generalization of the lifting scheme (4)- (7), we found by simulations that the coefficients of the lifting filters of an arbitrary order higher than 4 can be found according to (8) and the recursive formulas [8]:

$$p_2 = -\frac{p_1}{c_p},\ p_1 = p_1 - p_2,\ \ldots,\ p_{\tilde{N}} = -\frac{p_{\tilde{N}-1}}{c_p},\ p_{\tilde{N}-1} = p_{\tilde{N}-1} - p_{\tilde{N}}$$

$$u_2 = \frac{u_1}{c_u}, u_1 = u_1 - u_2,\ \ldots, u_N = \frac{u_{N-1}}{c_u}, u_{N-1} = u_{N-1} - u_{\tilde{N}} \tag{9}$$

where the parameters c_p, c_u controls the filter characteristics. This way, the lifting wavelet filters of an arbitrary order can be derived [8].

In the formula (8) the parameters b_p, b_u control the width of the transition bands and the new parameters c_p, c_u in (9) control the smoothness of the pass and stop bands to prevent the appearance of the lateral lobes: with greater values of b_p, b_u the values of c_p, c_u tend to be greater [8]. In practice, one can use predictor (4) and update filter (6) with $\tilde{N} = 6$, $N = 6$, $b_p = 20, b_u = 8$, $c_p = 6, c_u = 6$ to achieve narrow transition bands [8].

3 Proposed IIR Lifting Scheme

Considering generalized lifting scheme (4), (6) that these all-zeros systems can be modified to obtain rational transfer functions of a special form containing zeros and poles as following:

$$H_p(z) = \frac{1 + p_0\left(z + z^{-1}\right) + p_1\left(z^3 + z^{-3}\right) + \ldots + p_{\tilde{N}-1}\left(z^{2\tilde{N}-1} + z^{-2\tilde{N}+1}\right)}{1 + a_{2p}z^{-2} + a_{4p}z^{-4} + \ldots}, \tag{10}$$

$$H_u(z) = \frac{1 + H_p(z)\left\{u_0\left(z + z^{-1}\right) + u_1\left(z^3 + z^{-3}\right) + \ldots + u_{N-1}\left(z^{2N-1} + z^{-2N+1}\right)\right\}}{1 + a_{2u}z^{-2} + a_{4u}z^{-4} + \ldots}. \tag{11}$$

In (10), (11) the denominators contain only even powers of z because the outputs of predictor and update stages indirectly realize data subsampling (because of splitting (1)) and the presented transfer function are expressed in terms of input data sampling rate.

A specific condition to lifting predictor is that it must have a fixed gain to fulfill condition (7), i.e., to prevent bias in the output of the update filer at $\omega = 0$. This can be done introducing normalization by factor $1 - a_{2p} - a_{4p} - \ldots$ in (11):

$$H_u(z) = \frac{1 + H_p(z)\left\{u_0\left(z + z^{-1}\right) + u_1\left(z^3 + z^{-3}\right) + \ldots + u_{N-1}\left(z^{2N-1} + z^{-2N+1}\right)\right\}}{\left(1 - a_{2p} - a_{4p} - \ldots\right)\left(1 + a_{2u}z^{-2} + a_{4u}z^{-4} + \ldots\right)}. \tag{12}$$

Another problem arises when implementing inverse transform with IIR lifting. The wavelet analysis/synthesis filters must provide the perfect restoration of the original data that is especially important for lossless data compression. In the traditional dyadic wavelet decompositions/restorations technique, special care is took to design orthonormal filter banks where each filter satisfies Nyquist constraint $\left|H_k\left(e^{j\omega}\right)\right|_{\downarrow 2}=1$ [9]. In difference, the lifting scheme has a potential to design biorthogonal IIR wavelet filters in simpler way: in the restoration stage, one can use inverse predictor and inverse update filter that operates upon rearranging the input signal elements (wavelet coefficients) backward

$$\begin{aligned}\mathbf{s}^{BT}(n)&=[s(n-N),s(n-N-1),...,s(0)],\\ \mathbf{d}^{BT}(n)&=[d(n-N),d(n-N-1),...,d(0)].\end{aligned} \tag{13}$$

and then filtering them with the inverse filters

$$H_u(z)=\frac{1-H_p(z)\left\{u_0\left(z+z^{-1}\right)+u_1\left(z^3+z^{-3}\right)+..+u_{N-1}\left(z^{2N-1}+z^{-2N+1}\right)\right\}}{\left(1+a_{2p}+a_{4p}+...\right)\left(1-a_{2u}z^{-2}-a_{4u}z^{-4}-...\right)}, \tag{14}$$

$$H_p(z)=\frac{1-p_0\left(z+z^{-1}\right)-p_1\left(z^3+z^{-3}\right)-...-p_{\tilde{N}-1}\left(z^{2\tilde{N}-1}+z^{-2\tilde{N}+1}\right)}{1-a_{2p}z^{-2}-a_{4p}z^{-4}-...}, \tag{15}$$

for synthesis and next time performing rearranging of the data: $\{\cdot\}^B$.

Next, we want to proceed with integer calculus whereas it is possible. For this purpose, we use normalized coefficients a_{2p} , a_{2u} as in (8):

$$a_{ip}=\frac{A_{ip}}{256},\ a_{iu}=\frac{A_{iu}}{256}. \tag{16}$$

Taking into account all before mentioned results and restrictions, we can formulate the integer-to-integer IIR lifting steps as following.

Analysis stage:

- prediction:

$$d_i^{(k)}=d_i^{(k-1)}+\left\lfloor\frac{A_{2p}d_{i-2}^{(k-1)}+A_{2p}d_{i-4}^{(k-1)}+(b_p-128)\left(s_{i-1}^{(k-1)}+s_{i+1}^{(k-1)}\right)+b_p\left(s_{i-3}^{(k-1)}+s_{i+3}^{(k-1)}\right)+...}{256}\right\rfloor \tag{17}$$

- update:

$$s_i^{(k)}=s_i^{(k-1)}+\left\lfloor\frac{A_{2u}s_{i-2}^{(k-1)}+A_{4u}s_{i-4}^{(k-1)}}{256}+\frac{(64-b_u)\left(d_{i-1}^{(k)}+d_{i+1}^{(k)}\right)+b_u\left(d_{i-3}^{(k)}+d_{i+3}^{(k)}\right)+...}{256-A_{2p}-A_{4p}}\right\rfloor \tag{18}$$

Synthesis stage:
- inverse update:

$$s_i^{B(k)} = s_i^{B(k-1)} - \left\lfloor \frac{A_{2u}s_{i-2}^{B(k-1)} + A_{4u}s_{i-4}^{B(k-1)}}{256} + \frac{(64-b_u)\left(d_{i-1}^{B(k-1)} + d_{i+1}^{B(k-1)}\right) + b_u\left(d_{i-3}^{B(k-1)} + d_{i+3}^{B(k-1)}\right) + \ldots}{256 - A_{2p} - A_{4p}} \right\rfloor \tag{19}$$

- inverse prediction:

$$d_i^{B(k)} = d_i^{B(k-1)} - \left\lfloor \frac{A_{2p}d_{i-2}^{B(k-1)} + A_{2p}d_{i-4}^{B(k-1)} + (b_p - 128)\left(s_{i-1}^{B(k)} + s_{i+1}^{B(k)}\right) + b_p\left(s_{i-3}^{B(k)} + s_{i+3}^{B(k)}\right) + \ldots}{256} \right\rfloor \tag{20}$$

In formulas (17)-(18), $\lfloor \cdot \rfloor$ denotes the operation of rounding to the nearest lower integer value.

The coefficients $\{b_p\},\{b_u\}$ are that satisfy to (5), (7). Additionally, $\{b_p\},\{b_u\}$, $\tilde{N}, N$ and especially $\{A_p\},\{A_u\}$ are adjusted in such a manner that the filters (17), (18) match to the spectral properties of the image data to minimize the well known first order entropy of the wavelet coefficients

$$\min_{\{b_p\},\{b_u\},\tilde{N},N,\{A_p\},\{A_u\}} \left\{ H(d) = -\sum_i p_i \log(p_i) \right\} \tag{21}$$

where p_i denotes the probability of the different values of wavelet coefficients d. The problem of optimization can be formulated as the problem to minimize the following errors:

- the square error of prediction

$$\varepsilon_p = \sum_i \left[d_i^{(k)}\right]^2 ; \tag{22}$$

- the square error of “update first” prediction

$$\varepsilon_u = \sum_i \left[\varepsilon_i^u\right]^2 , \tag{23}$$

where $\varepsilon_i^u = s_i^{(k-1)} - \left\lfloor \frac{A_{2u}s_{i-2}^{(k-1)} + A_{4u}s_{i-4}^{(k-1)}}{256} + \frac{(64-b_u)\left(\tilde{d}_{i-1}^{(k)} + \tilde{d}_{i+1}^{(k)}\right) + b_u\left(\tilde{d}_{i-3}^{(k)} + \tilde{d}_{i+3}^{(k)}\right) + \ldots}{256 - A_{2p} - A_{4p}} \right\rfloor$,

and $\tilde{d}_i^{(k)} = d_i^{(k-1)} - \left\lfloor \frac{A_{2p}d_{i-2}^{(k-1)} + A_{2p}d_{i-4}^{(k-1)} + (b_p - 128)\left(s_{i-1}^{(k-1)} + s_{i+1}^{(k-1)}\right) + b_p\left(s_{i-3}^{(k-1)} + s_{i+3}^{(k-1)}\right) + \ldots}{256} \right\rfloor$

is the output of “update first” low-pass filter.

Thus, one can obtain the optimal solution finding

$$\min_{\substack{\{b_p\},\{b_u\},\\ \{A_p\},\{A_u\},\\ \{c_p\},\{c_u\}}} \{\varepsilon_p + \varepsilon_u\}$$

at each level of decomposition. Unfortunately, it is difficult to obtain an analytical solution for the optimal $\{b_p\},\{b_u\}$, $\tilde{N}, N$, $\{A_p\},\{A_u\}$ values due to complexity of the expressions (22), (24).

4 Experimental Results

The described in the previous section algorithm were tested on a set of 512x512 standard images "Lena", "Baboon", "Barbara", "Boats", "Goldhill", "Peppers", "Bridge" shown in Fig. 1 (these images are available, for example, at http://sipi.usc.edu/database/).

Fig. 1. Set of standard test images: "Lena", "Baboon", "Barbara", "Boats", "Goldhill", "Peppers", "Bridge" v

Table 1 presents the entropy values in bits per pixel (bpp) obtained for these images by applying standard lifting decomposition (1) – (3) and CDF(1,1) wavelet (Haar wavelet) with $\tilde{N}=1$, $N=1$, $a=0$, $b=0$, CDF(2,2) wavelet with $\tilde{N}=2$, $N=2$, $a=16$, $b=8$ (this wavelet is used by JPEG2000 for lossless image compression), and IIR lifting (17)-(20) with the same FIR parameters and various $\{A_p\},\{A_u\}$ values. The values of $\{A_p\},\{A_u\}$ are those that minimize the first order entropy of the wavelet coefficients in the first level of decompositions, in higher levels they were chosen to be 0.

Table 2 presents the entropy values in bits per pixel (bpp) obtained for the test images by applying generelazid lifting decomposition (4), (6) and IIR lifting (17)-(20). The values of FIR part of the lifting scheme were varied and the values $\{A_p\},\{A_u\}$ were the same parameters as in the previous simulations (see Table 1).

Analyzing the simulation results presented in Table 1 and Table 2, one can conclude that the proposed IIR lifting transform performs better, providing lower entropy values for all test images in comparison to the FIR lifting. Increasing FIR lifting orders $\tilde{N}, N$ and varying FIR coefficients b_p, b_u, c_p, c_u without using IIR coefficients ($\{A_p\} = \vec{0}, \{A_u\} = \vec{0}$), one can obtain higher data compression. The difference between FIR and IIR performance is small sometimes (for example, for Lena image), but in all cases, the IIR technique gives the best compression results.

Table 1. Entropy values in bpp for different techniques, $\tilde{N} = 2$, $N = 2$, $b_p = 16$, $b_u = 8$ in cases of CDF(2,2) and IIR lifting with the correspondent $\{A_p\}, \{A_u\}$ values

Technique	Image						
	Baboon	Lena	Barbara	Boat	Bridge	Peppers	Goldhill
CDF(1,1) lifting	6.163	4.405	5.087	5.014	3.789	4.715	4.898
CDF(2,2) Lifting	6.137	4.361	4.940	4.976	3.793	4.711	4.885
IIR Lifting	**6.128**	**4.355**	**4.914**	**4.966**	**3.792**	**4.686**	**4.871**
A_{2p}	19	11	28	-11	0	-26	15
A_{4p}	7	-4	11	-3	0	9	16
A_{2u}	9	3	8	-19	3	5	-2
A_{4u}	-8	0	-8	-3	-5	3	-7

Table 2. Entropy values in bpp for different techniques, $\tilde{N} = 8$, $N = 8$, $\{A_p\}, \{A_u\}$ are those from Table 1, b_p, b_u, c_p, c_u were varied to achieve the minimum bpp

Technique	Image						
	Baboon	Lena	Barbara	Boat	Bridge	Peppers	Goldhill
Generalized lifting	6.134	4.359	4.825	4.972	3.792	4.701	4.885
IIR Lifting	**6.125**	**4.351**	**4.817**	**4.963**	**3.791**	**4.678**	**4.869**
b_p	20	20	29	18	11	9	15
b_u	13	9	18	11	8	11	4
c_p	6	6	2	5	4	4	6
c_u	5	6	2	6	9	6	3

5 Conclusions

The novel algorithm of data-dependent DWT based on the generalized IIR lifting scheme is presented. The proposed algorithm requires only four additional integer sums and one floating point multiplication per pixel in comparison to the standard lifting decomposition. The presented previous results show that the derived algorithm provides lossless image compression and the highest data compression rate comparing to the standard wavelet lifting technique. The simulations were performed for the case when the IIR filtering is applied for the first level of decomposition only and at higher levels the generalized FIR lifting was used, because of the difficulties dealing with finding the optimal filter coefficients. One can expect even better energy compaction, and, thus, higher compression rate with the presented algorithm optimizing the filter coefficients for each wavelet decomposition. This aspect of global/local optimization according to (23), (24) to find the optimal filter coefficients b_p, b_u, c_p, c_u, $\{A_p\}, \{A_u\}$ is a subject of future work.

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References

1. S.G.Mallat, A theory for multiresolution signal decomposition: The wavelet representation, *IEEE Trans. Pattern Recognition. Machine Intell.*, *Vol. 11*, No. 7, p.p.674-693, 1989.
2. Martin Vetterli, Multi-dimensional sub-band coding: some theory and algorithms, *Signal Processing*, *Vol. 6*, pp.97-112, 1984.
3. I.Daubechies, Orthonormal bases of compactly supported wavelets, *Commun. Pure Appl. Math.*, *Vol. 41*, p.p. 909-996, Nov. 1998.
4. W.Sweldens, The lifting scheme: A new philosophy in biorthogonal wavelet constructions, *Wavelet Applications in Signal and Image Processing III*, *A.F.Laine and M.Unser, editors,Proc. SPIE 2569,* p.p. 68-79, 1995.
5. R.Claypoole, R.Baraniuk, and R.Nowak, Adaptive wavelet transforms via lifting, *Proc. IEEE Int. Conf. Acoust., Speech, and Signal Proc.*, May 1998.
6. G. Piella and H. J. A. M. Heijmans, "An adaptive update lifting scheme with perfect reconstruction," *Proc. ICIP'01*, pp. 190 - 193, October 2001.
7. G.C.K. Abhayaratne, "Spatially adaptive wavelet transforms: An optimal interpolation approach", Third International Workshop on Spectral Methods and Multirate Signal Processing (SMMSP) 2003, pp. 155-162, Barcelona, Spain, 12-13 Sept. 2003.
8. Oleksiy Pogrebnyak, Pablo Manrique Ramírez "Adaptive wavelet transform for image compression applications" Proc. SPIE Vol.5203, Applications of Digital Image Processing XXVI, Andrew G. Tescher Chair/Editor, 5- 8 August 2003, San Diego, USA. p.p. 623-630, ISBN 0-8194-5076-6 , ISSN: 0277-786X.
9. P. P. Vaidyanathan and S. Akkarakaran, ``A review of the theory and applications of principal component filter banks," Applied and Computational Harmonic Analysis (invited paper), vol. 10, no. 3, pp. 254-289, May 2001.
10. J. O. Chapa and R. M. Rao, "Algorithms for designing wavelets to match a specified signal," *IEEE Trans. Signal Processing,* vol. 48, pp. 3395 - 3406, December 2000.
11. P. P. Vaidyanathan, ``Theory of optimal orthonormal subband coders", IEEE Trans. Signal Proc., vol. SP. 46, pp. 1528--1543, June 1998.

12. P. Moulin, M. Anitescu, and K. Ramchandran, "Theory of rate-distortion-optimal, constrained filterbanks--Application to IIR and FIR biorthogonal designs," *IEEE Trans. Signal Processing,* vol. 48, pp. 1120 - 1132, April 2000
13. Hoon Yoo and Jechang Jeong, A Unified Framework for Wavelet Transform Based on The Lifting Scheme, *Proc. of IEEE International Conference on Image Processing ICIP2001*, Tessaloniki, Greece, October 7-10, p.p.793-795, 2001.