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EDITORIAL

Special issue on "Applied bilevel programming"

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Introduced in the 1970s, the term *bilevel programming* refers to the paradigm where an *upper level* agent optimizes, with respect to a vector x_1, over the solution set $S(x_1)$ of a *lower level* program parameterized in x_1, i.e.,

$$\begin{aligned} \min_{x_1,x_2} \quad & f_1(x_1, x_2) \\ \text{subject to} \quad & (x_1, x_2) \in X_1, \;\; x_2 \in S(x_1), \end{aligned}$$

where $S(x_1) = \arg\min_{x_2 \in X_2(x_1)} f_2(x_1, x_2)$.

This situation arises whenever the upper level agent embeds within its optimization process the reaction of the lower level to its course of actions, and arises in fields as diverse as economics, telecommunications, transportation, engineering, or chemistry. In the past decade, the field has developed into an important area of mathematical programming, with several articles devoted to theory, models, and computational methods.

Bilevel programs are closely related to leader–follower games, for which the Stackelberg duopoly game is the quintessential example. They are also related to the class of *mathematical programs with equilibrium constraints* (*MPECs*), where the elements of the set $S(x_1)$ are equilibrium states of a system parameterized in x_1, frequently expressed in terms of the variational inequality

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$$S(x_1) = \{x_2 \in X_2(x_1) : \langle F_2(x_1, x_2), x_2 - x_2' \rangle \leq 0 \quad \forall x_2' \in X_2(x_1)\}.$$

Whenever the set $X_2(x_1)$ is a singleton $x_2(x_1)$, bilevel programs and MPECs can be recast as the 'standard' optimization problem

$$\min_{(x_1, x_2) \in X_1} f_1(x_1, x_2(x_1)).$$

Unfortunately, the latter is usually nondifferentiable and nonconvex, and its domain could even be disconnected, making the characterization and computation of solutions difficult, even locally.

Alternatively, if conditions ensuring the applicability of Karush–Kuhn–Tucker conditions apply, the bilevel program (or MPEC) can be expressed as a single-level program involving primal-dual complementarity constraints. Such reformulation outlines the combinatorial nature of the problem, which must be addressed if a global solution is sought for.

The suitability of bilevel programs for the modeling of practical situations, as well as the challenges posed by their numerical resolutions, has led to significant advances in the field. The current issue of EJCO focuses on two specific applications and an algorithmic issue, leaving aside the more theoretical aspects.

The papers address very different topics. The first one is concerned with an auction model aimed at optimizing supply and consumption in an electricity market through a pricing mechanism that considers explicitly a large number of technical constraints. An important feature of the model is that dual variables of the lower level constraints explicitly enter the upper level's objective. This work illustrates the growing interest for bilevel programming in energy modeling, as can be observed in the paper's bibliography.

The second paper is concerned with robust data fitting, a research area whose growth is correlated with that of 'big data'. Its authors show how the problem of parameter selection associated with 'support vector machines' can be formulated as a bilevel program. They design a global optimization algorithm for its solution, and provide evidence that it is competitive with alternative approaches on realistic instances.

In the third paper, the authors investigate the properties of an approximation scheme for mathematical programs with complementarity constraints, a class of problems that is equivalent to MPECs. They show that, under mild conditions, the iterative algorithm possesses strong local convergence properties. These theoretical results are validated by numerical experiments performed on standard benchmarks taken from MacMPEC repository.

The three papers are witnesses of the variety of situations that fit the bilevel framework, and our hope is that this special issue trigger the interest of further research in bilevel programming, especially from the application and computational points of view.

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