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A Contractive Sliding-mode MPC Algorithm for Nonlinear Discrete-time Systems

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Abstract: This paper investigates a sliding-mode model predictive control (MPC) algorithm with auxiliary contractive sliding vector constraint for constrained nonlinear discrete-time systems. By adding contractive constraint into the optimization problem in regular sliding-mode MPC algorithm, the value of the sliding vector is decreased to zero asymptotically, which means that the system state is driven into a vicinity of sliding surface with a certain width. Then, the system state moves along the sliding surface to the equilibrium point within the vicinity. By applying the proposed algorithm, the stability of the closed-loop system is guaranteed. A numerical example of a continuous stirred tank reactor (CSTR) system is given to verify the feasibility and effectiveness of the proposed method.

Keywords: Model predictive control (MPC), sliding mode, contractive constraint, discrete-time systems, nonlinear systems.

1 Introduction

Model predictive control (MPC), also called moving horizon control (MHC) and receding horizon control (RHC), is the most attractive control strategy for systems with input and state constraints. The current control action of MPC is obtained by solving a finite horizon optimization problem at each sampling time, and the first one is applied to the plant. At the next sampling time, the same procedure is repeated.

Linear MPC (LMPC) is a control scheme for linear systems, which has been studied extensively[1]. However, most of the practical systems have nonlinearities. Hence, the nonlinear MPC (NMPC) algorithms should be applied instead of LMPC strategies in order to get the high quality of control performance. Because of the inherent difficulties in analyzing nonlinear control systems, NMPC theory is far from perfect and many challenges still exist, such as stability, robustness, computational burden, etc.[2−5]

The major difficulty of NMPC is guaranteeing the closed-loop stability. In order to guarantee the closed-loop stability, various stability constraints have been proposed. The simplest approach is to add a terminal equality constraint into the optimization problem[6]. It requires that the state exactly converges to zero in finite steps. It is conservative, and the optimization problem may become infeasible. For relaxation, the terminal inequality constraint is applied, where terminal state is enforced to a region which includes equilibrium point in its interior, instead of a point (equilibrium point). By combining the terminal cost function with terminal inequality constraint, Chen and Allgöwer[7] proposed a quasi-infinite NMPC strategy, which can get the infinite horizon control performance by minimizing the upper bound of infinite horizon cost functions. Oliveira and Morari[8] proposed the contractive constraint, which adds a terminal contractive constraint in the optimization problem to guarantee the system stability. In order to prove the closed-loop stability, a block optimization strategy is adopted. Xie[9] presented the first state contractive NMPC algorithm, in which the contractive constraints are enforced on the one-step ahead predicted state. Sun et al.[10] presented another contractive NMPC algorithm, which adopts a time-varying implementation horizon confirmed by solving an appropriate optimization problem.

As an important branch of variable structure control[11−13], sliding mode control (SMC) is characterized by switching the control law during the evolution of the state, and enforcing the states to the predefined asymptotic stable sliding surface. We call the control algorithm which combines MPC with SMC the sliding-mode MPC (SM-MPC). Parte et al.[14] designed a generalized predictive control (GPC) method based on sliding mode controller. Xiao et al.[15] addresed a similar approach, where the model algorithm control (MAC) is used. Zhou et al.[16] presented an SM-MPC algorithm for systems with state space model, which takes sliding vector as a new variable, and stabilizes it by dual-mode MPC.

Inspired by [16], this paper takes sliding vector as a new variable, and stabilizes it by MPC algorithm with extra contractive sliding vector constraint for constrained nonlinear systems. This makes the sliding vector contract to zero step by step. It implies that the system state implicitly satisfies the reaching condition. The proposed algorithm improves the overall feasibility, and avoids the switching between inner mode controller and outer mode controller. The closed-loop stability is guaranteed if asymptotic stable sliding mode is predesigned.

This paper is organized as follows. Section 2 describes the problem to be studied. Section 3 presents the new contractive SM-MPC. The stability is discussed and proved in Section 4, which is mainly inspired by the method in [8]. In Section 5, we apply the proposed algorithm to a practical system and verify the feasibility of the proposed algorithm. Note that, this paper makes a tentative research on the SM-MPC algorithm with contractive constraint, and any further improvement is encouraged for the readers.

Notations. $\mathbf{R}$ denotes the set of real numbers. The Euclidean norm of a vector x is denoted as $||x||$, and $||x||_{\hat{P}}$

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is the weighted Euclidean norm with respect to x, where $\hat{P}$ is the weighting matrix. N stands for the prediction horizon. The mathematical function int (x) gets the largest integer not greater than x. $k_j = j \times N$ for any $j \in \mathbf{Z}_+$ is the set of nonnegative integers.

2 Problem statement

Consider the time-invariant nonlinear dynamic system described in the following state space equation:

$$x(k+1) = f(x(k), u(k)), \quad x(0) = x_0, \quad k \geqslant 0 \tag{1}$$

where $x(k) \in \mathbf{R}^n, u(k) \in \mathbf{R}^m$ are the state and input vectors at sampling time k, and the system state and input are constrained by

$$x(k) \in X \subseteq \mathbf{R}^n,\ u(k) \in U \subseteq \mathbf{R}^m. \tag{2}$$

The following assumptions hold throughout the paper[7].

Assumption 1. $f : \mathbf{R}^n \times \mathbf{R}^m \rightarrow \mathbf{R}^n$ is twice continuously differentiable and $f(0,0) = 0$. Thus, $0 \in \mathbf{R}^n$ is an equilibrium point of the system with $u = 0$.

Assumption 2. $U \subseteq \mathbf{R}^m$ and $X \subseteq \mathbf{R}^n$ are compact, convex, and contain the origin as an interior point.

Remark 1. In [7], besides Assumptions 1 and 2, it also needs to assume that the pair (A, B) is stabilizable, where $A := \frac{\partial f}{\partial x}(0,0)$ and $B := \frac{\partial f}{\partial u}(0,0)$ lead to the Jacobian linearization of the system (1) at the origin:

$$x(k+1) = Ax(k) + Bu(k). \tag{3}$$

For the nonholonomic systems, like cars, mobile robots, etc., the pair (A, B) cannot be stabilized, so the MPC algorithm in [7] cannot be implemented in these systems. But in our algorithm, this assumption is not needed, i.e., the proposed algorithm can be used to control the nonholonomic systems.

For the nonlinear discrete-time system (1), the regular NMPC algorithm[17] is to solve the optimization problem described as

$$P_1(k, x, N) : \arg \min_{u(k|k), \cdots, u(k+N-1|k)} J(x(k), u(\cdot)) \tag{4}$$

$$\text{s.t.} \quad x(k+i+1|k) = f(x(k+i|k), u(k+i|k)) \tag{5}$$

$$x(k+i|k) \in X, i = 0, \cdots, N-1 \tag{6}$$

$$u(k+i|k) \in U, i = 0, \cdots, N-1 \tag{7}$$

$$x(k+N|k) \in \Omega \tag{8}$$

where

$$\begin{aligned} J(x(k), u(\cdot)) = \sum_{i=0}^{N-1} \left\{ ||x(k+i|k)||_Q^2 + ||u(k+i|k)||_R^2 \right\} + \\ ||x(k+N|k)||_P^2 \end{aligned} \tag{9}$$

$x(k+i|k)$ is the prediction of x at the future time $k+i$, predicted at time k. Ω is the terminal constraint set near the equilibrium point. The corresponding weighting matrices Q, P, R are positive definite.

In the sequel, before introducing the SM-MPC algorithm, some basic definitions and designing procedures about sliding mode control are reviewed.

Definition 1 (Sliding mode)[18]. Sliding mode, also called sliding motion, can be defined as the evolution of the states of a system confined to a specified sub-manifold of the state space with stable dynamics. Considering a special case of the system with linear sliding surface $s = Cx = 0$, where $C \in \mathbf{R}^{m \times n}$, we can define a null space as

$$S := \mathcal{N}(C) = \{x | s = Cx = 0,\ C \in \mathbf{R}^{m \times n},\ x \in \mathbf{R}^n\}. \tag{10}$$

Then, the sliding mode for this special case is the motion of state in $\mathcal{N}(C)$.

Definition 2 (Quasi-sliding mode)[19]. The quasi-sliding mode is the motion of state in the Δ vicinity of sliding surface, such that the system state, once entering this band, never leaves it. Define

$$S_\Delta := \{x \big| \, ||s|| = ||Cx|| \leqslant \Delta\} \tag{11}$$

where the positive constant $2 \times \Delta$ represents the bandwidth of quasi-sliding mode.

Definition 3 (Equivalent control $\boldsymbol{u}_{\mathbf{eq}}$[20]). The equivalent control is derived by solving

$$\begin{cases} x(k+1) = f(x(k), u(k)) \\ s(k+1) = 0 \end{cases} \tag{12}$$

which is explicit with respect to state variables, i.e., $u_{\rm eq} = \kappa(x)$. For simplicity, we take a linear system with linear sliding surface as a special example. Solving

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ Cx(k+1) = 0 \end{cases} \tag{13}$$

yields $u_{\rm eq}(k) = -(CB)^{-1}CAx(k)$.

For the nonlinear system (1), the procedure to design a sliding mode controller can be listed as follows[20]:

1) Choose a sliding vector $s(x)$ with stable sliding mode. Sliding vector can be chosen as linear one with respect to state, e.g., $s = Cx$. The parameter C can be easily confirmed for linear systems.

2) Calculate the state-feedback control action under the reaching condition. With the predesigned asymptotic stable sliding surface $s = 0$, we can get the explicit state-feedback control law which meets the reaching condition $s\dot{s} < 0$ (continuous-time system) or $||s(k+1)|| < ||s(k)||$ (discrete-time system).

Combining the sliding mode with MPC algorithm, Zhou et al.[16] proposed the following algorithm:

$$P_2(k, s, N) :\ \arg \min_{u(k|k), \cdots, u(k+N-1|k)} J(s(k), u(\cdot)) \tag{14}$$

$$\text{s.t. } (5), (6), (7), s(k+N|k) \in S. \tag{15}$$

with

$$\begin{aligned} J(s(k), u(\cdot)) = \sum_{i=0}^{N-1} \left\{ ||s(k+i|k)||_Q^2 + ||u(k+i|k) - \right. \\ \left. u_{\rm eq}(k+i|k)||_R^2 \right\} + ||s(k+N|k)||_{\tilde{P}}^2 \end{aligned} \tag{16}$$

where $\tilde{P}$ is a positive definite weighting matrix.

In this algorithm, the sliding vector s is taken as a new state. Then, it is stabilized by the dual-mode MPC strategy, i.e.,

$$u(k) = \begin{cases} u^*(k|k), & x(k) \in S \\ u_{\rm eq}(k), & x(k) \notin S \end{cases} \tag{17}$$

where $u^*(k|k)$ is the first input in the solution of optimization problem (16), and $u_{\rm eq}(k)$ is the equivalent control.

It is difficult to calculate the weighting matrix $\tilde{P}$ in the terminal cost function $||s(k+N|k)||^2_{\tilde{P}}$ for the nonlinear system. In the sequel, in order to get a more feasible SM-MPC algorithm, we will provide a novel SM-MPC with contractive constraints.

3 Contractive SM-MPC algorithm

As seen from [8], adding auxiliary contractive constraint to the optimization problem is an effective way to guarantee closed-loop stability for MPC algorithms. Therefore, in this paper, the contractive constraint is attached to the regular SM-MPC so that the contractive SM-MPC algorithm is presented. The contractive SM-MPC algorithm enforces the variable of sliding vector to decay to zero or other equilibrium point step by step.

The proposed contractive SM-MPC algorithm solves the following optimization problem:

$$P_3(k,s,N):\ \arg\min_{u(k|k),\cdots,u(k+N-1|k)} J(s(k),u(\cdot)) \tag{18}$$

$$\text{s.t.}\ \ (5),(6),(7),$$

$$||s(k+N|k)||_{\hat{P}} \leqslant \rho\ ||s(k|k)||_{\hat{P}},\ \rho\in[0,1) \tag{19}$$

where the objective function $J(s(k),u(\cdot))$ is defined as

$$J(s(k),u(\cdot)) = \sum_{i=1}^{N}\left\{||s(k+i|k)||^2_Q + ||u(k+i-1|k) - u_{\rm eq}(k+i-1|k)||^2_R\right\}. \tag{20}$$

The specific procedure of contractive SM-MPC can be described as follows.

Algorithm 1. Contractive SM-MPC algorithm

Step 1. Measure the initial value $x(0)$. Give the sampling time $T>0$, the prediction horizon $N\in\mathbf{Z}_+$, the contraction rate $\rho\in[0,1)$, the constraint sets X and U, and the weighting matrices $Q>0$, $R>0$, $\hat{P}>0$.

Step 2. Set $k=0$.

Step 3. Measure the value of state $x(k)$ at time k. Solve the optimal control problem $P_3(k,s,N)$, and get the control sequence $\{u(k|k),\cdots,u(k+N-1|k)\}$.

Step 4. For the future N steps: $k,\cdots,k+N-1$, apply the corresponding precalculated control action in the control sequence to the plant at the relevant time.

Step 5. Set $k=k+N$. Go to Step 2.

Remark 2. Compared with (14), the auxiliary contractive constraint (19) is adopted in the optimization problem instead of the terminal constraint in (17). Unlike the dual-mode strategy, this algorithm avoids the switching of the controller. Moreover, the computational burden is alleviated since the block optimization strategy is utilized, i.e., the optimization problem (18) will be solved at every N steps.

4 Stability analysis

In this section, the stability of the contractive SM-MPC algorithm will be proved. Before we give the main results, the following assumptions should be made firstly[18].

Assumption 3 (The motion of sliding mode is stable). The predesigned sliding surface is a stable subspace of the state space. When the system state is steered into the sliding surface, it will keep in it by applying the calculated control sequence, and the system state will decay to the equilibrium point asymptotically.

Remark 3. For the proposed algorithm, Assumption 3 is the most essential point for nonlinear systems to guarantee the closed-loop stability. The state trajectory can be divided into two parts. The first part is the reaching mode, it is the length of state trajectory which starts from $k=0$ to the time when state begins to enter into the sliding surface. The second part is the sliding mode, which is the trajectory of state in the sliding surface. Thus, we need to assume that system in the sliding mode phase has guaranteed closed-loop stability.

Assumption 4. For $k\in[k_j,k_{j+1}]$, there exists a constant parameter $\beta\in(0,\infty)$ so that the transient sliding vector $s(k)$ satisfies the inequality $||s(k)||_{\hat{P}}\leqslant\beta||s(k_j)||_{\hat{P}}$.

Remark 4. Assumption 4 assumes that the sliding vector, from sampling time k to sampling time $k+N$, is bounded. For the application on continuous-time systems, the magnitude of sliding vector cannot be infinity if the sampling time is small enough. Therefore, Assumption 4 is reasonable.

Assumption 5 (Feasibility of the optimization problem). There exists a constant parameter $\sigma\in(0,\infty)$ such that for all $x(k_j)\in B_\sigma:=\{x\in\mathbf{R}^n|\,||x||\leqslant\sigma\}$, under constraints (5)−(7), the optimization problem (18) with cost function (16) is feasible for all $k\in\mathbf{Z}_+$. In other word, for all $x(k_j)\in B_\sigma$, we can find a value of $\rho\in[0,1)$ so that all constraints on state and input variables will be satisfied and the optimization problem is feasible.

Remark 5. For sliding vector s, there is always a bound on it. Hence, it is reasonable in Assumption 4 to assume that the sliding vector has its bound in every interval of $[k_j,k_{j+1}]$, $j=0,1,\cdots$. In Assumption 5, the feasibility of optimization problem (18) is assumed within a certain region. Since the parameter $\rho\in[0,1)$, one can always find a value of ρ which makes the optimization feasible.

Theorem 1. Suppose Assumptions 1−3 are satisfied. Give rate $\rho\in[0,1)$. Let the parameters $\beta\in(0,\infty)$ and $\sigma\in(0,\infty)$ satisfy Assumptions 4 and 5, respectively. Then, for any $x_0\in B_\sigma$, the system will be stable if the resulting trajectory of sliding vector satisfies the following inequality:

$$||s(k)||_{\hat{P}}\leqslant\beta||s(0)||_{\hat{P}}\mathrm{e}^{-(1-\rho)[(k/N)-1]},\quad \forall k\in\mathbf{Z}_+. \tag{21}$$

Proof. For $k\in[k_j,k_{j+1}]$, the following inequalities can be easily deduced from (19) and Assumption 4:

$$||s(k_j)||_{\hat{P}}\leqslant\rho||s(k_{j-1})||_{\hat{P}}\leqslant\cdots\leqslant\rho^j||s(0)||_{\hat{P}}$$

$$||s(k)||_{\hat{P}}\leqslant\beta||s(k_j)||_{\hat{P}}\leqslant\beta\rho^j||s(0)||_{\hat{P}}$$

where $j=0,1,\cdots$.

Due to $\rho\in[0,1)$ and $k\in\mathbf{Z}_+$, we have

$$\mathrm{e}^{(\rho-1)}-\rho\geqslant0\Leftrightarrow\rho^k\leqslant\mathrm{e}^{-(1-\rho)k}.$$

Then,

$$||s(k_j)||_{\hat{P}}\leqslant||s(0)||_{\hat{P}}\mathrm{e}^{-(1-\rho)j}$$

and

$$||s(k)||_{\hat{P}} \leqslant \beta ||s(0)||_{\hat{P}} \mathrm{e}^{-(1-\rho)j}. \tag{22}$$

For $\forall k \in \mathbf{Z}_+$, from (22), we can get

$$||s(k)||_{\hat{P}} \leqslant \beta ||s(0)||_{\hat{P}} \mathrm{e}^{-(1-\rho)\times \mathrm{int}(\frac{k}{N})}$$

Due to $\mathrm{int}(\frac{k}{N}) \geqslant (\frac{k}{N}) - 1$, we have

$$\beta \mathrm{e}^{-(1-\rho)\times \mathrm{int}(\frac{k}{N})} \leqslant \beta \mathrm{e}^{-(1-\rho)[(\frac{k}{N})-1]}$$

Thus, we finally have (21). □

From Assumption 3, we can see that the sliding surface has the pre-specified asymptotic stable sliding mode. When the system state is forced into the sliding surface, the system state will slide to the equilibrium point. Overall, this paper is only an attempt to tackle the SM-MPC. Readers may improve this result to a more general case.

Remark 6. For the continuous-time systems, under Assumption 3, the system will have asymptotically stable sliding mode. Thus, if the system state is enforced into the sliding surface by the calculated control moves, then it will slide to the equilibrium point. From (21), we can see that sliding vector s will decrease exponentially to zero, i.e., the system state will reach the sliding surface. Ultimately, the closed-loop stability is proved and the system is stable if condition (21) is satisfied. Since the controlled system is discrete, only quasi-sliding mode can be formed, i.e., the system state can only be stabilized into a vicinity of the equilibrium point.

5 Numerical simulation

The proposed contractive SM-MPC algorithm is applied to the continuous stirred tank reactor (CSTR) system which is borrowed from [21]. The corresponding model is described by

$$\begin{cases} \dot{C}_A = \dfrac{q}{V}(C_{Af} - C_A) - k_0 \mathrm{e}^{-\frac{E}{RT}} C_A \\ \dot{T} = \dfrac{q}{V}(T_f - T) + \dfrac{(-\Delta H)}{\rho C_p} k_0 \mathrm{e}^{-\frac{E}{RT}} C_A + \\ \quad \dfrac{UA}{V\rho C_p}(T_c - T)] \end{cases} \tag{23}$$

where C_A is the concentration of the reactant A in the reactor, T is the reactor temperature, and T_c is the temperature of the coolant stream. The constraints are $280\,\mathrm{K} \leqslant T_c \leqslant 370\,\mathrm{K}$, $280\,\mathrm{K} \leqslant T \leqslant 370\,\mathrm{K}$, $0 \leqslant C_A \leqslant 1\,\mathrm{mol/L}$.

Choose the unstable equilibrium as $C_A^{\mathrm{eq}} = 0.5\,\mathrm{mol/L}$, $T^{\mathrm{eq}} = 350\,\mathrm{K}$, and $T_c^{\mathrm{eq}} = 300\,\mathrm{K}$. The corresponding parameters for nominal operation are $q = 100\,\mathrm{L/min}$, $T_f = 350\,\mathrm{K}$, $V = 100\,\mathrm{L}$, $\rho = 1000\,\mathrm{g/L}$, $C_p = 0.239\,\mathrm{J/g\cdot K}$, $\Delta H = -5 \times 10^4$ J/mol, $E/R = 8750\,\mathrm{K}$, $k_0 = 7.2 \times 10^{10}\,\mathrm{min}^{-1}$, and $UA = 5 \times 10^4\,\mathrm{J/min\cdot K}$.

The objective is to regulate C_A and T by manipulating T_c. Thus, define the state vector $x = [x_1,\ x_2]^{\mathrm{T}} = [C_A - C_A^{\mathrm{eq}},\ T - T^{\mathrm{eq}}]^{\mathrm{T}}$ and the manipulated input $u = T_c - T_c^{\mathrm{eq}}$. By substituting each parameter with specific value, (23) can be discretized with sampling interval $T_s = 0.05\,\mathrm{min}$ as

$$\begin{cases} x_1(k+1) = \\ \quad x_1(k) + T_s(0.5 - x_1(k) - 7.2 \times 10^{10} \times \\ \quad \mathrm{e}^{-\frac{8750}{x_2(k)+350}}(x_1(k) + 0.5)) \\ x_2(k+1) = \\ \quad x_2(k) + T_s(-x_2(k) + 1.5063 \times 10^{13} \times \\ \quad \mathrm{e}^{-\frac{8750}{x_2(k)+350}}(x_1(k) + 0.5)) + \\ \quad 2.0921 \times 10^6 \times (u(k) - x_2(k) - 50)). \end{cases} \tag{24}$$

Let

$$Q = R = \hat{P} = \tilde{P} = 1. \tag{25}$$

The sliding vector s is designed as

$$s(k) = cx_1(k) + x_2(k) \tag{26}$$

where $c = -10$.

From (12), the equivalent control is

$$\begin{aligned} u_{\mathrm{eq}}(k) = &-\frac{1}{2.0921 \times 10^6 \times T_s} \{-T_s - 1.04605 \times 10^8 + \\ &2(T_s - 1)x_1(k) + (1 - T_s - T_s \times 2.0921 \times 10^6) \times \\ &x_2(k) + T_s(1.5135 \times 10^{13}) \times \mathrm{e}^{-\frac{8750}{x_2(k)+350}} \times \\ &(x_1(k) + 0.5)\}. \end{aligned} \tag{27}$$

Choose $N = 5$, $\rho = 0.6$, and initial value $x(0) = [-0.4, -1.2]^{\mathrm{T}}$ in Algorithm 1. In order to clarify the advantage of the proposed algorithm, we choose the same sliding vector, prediction horizon and initial value of states as in Algorithm 1 for the regular SM-MPC algorithm in [16]. Then, the simulation results of each algorithm are shown in Table 1 and Figs. 1−2.

Table 1 Comparison of computation time

Algorithm	Computation time
Contractive SM-MPC	1.9 s
Regular SM-MPC[16]	4.3 s

It can be seen from Fig. 1 that the system, controlled by the proposed contractive SM-MPC algorithm, is asymptotically stable, and all the hard constraints on state and input variables are not violated. The sliding vector s in Fig. 1 has decayed to zero in finite time. From Figs. 1−2 and Table 1, we know that the contractive SM-MPC algorithm has better control performance than regular SM-MPC algorithm. Moreover, the computation time of contractive SM-MPC is significantly decreased.

6 Conclusions

In this paper, a contractive SM-MPC algorithm which adds a contractive sliding vector constraint into the regular SM-MPC optimization problem is proposed. With the predesigned stable sliding surface, the system state decays to the vicinity of the equilibrium point if the reaching condition has been satisfied. Hence, we take sliding vector as a new variable and then stabilize it by MPC algorithm with contractive constraint. Compared with the existing SM-MPC algorithm, the presented contractive SM-MPC algorithm has less conservativeness and lighter computational burden. The result in this paper relies heavily on the assumptions made on the nonlinearity in the vicinity of the sliding surface. Readers are encouraged to improve the result to the more general cases. Indeed, it is difficult to design the sliding surface with stable sliding mode for a general nonlinear system. We believe that this research topic is promising.

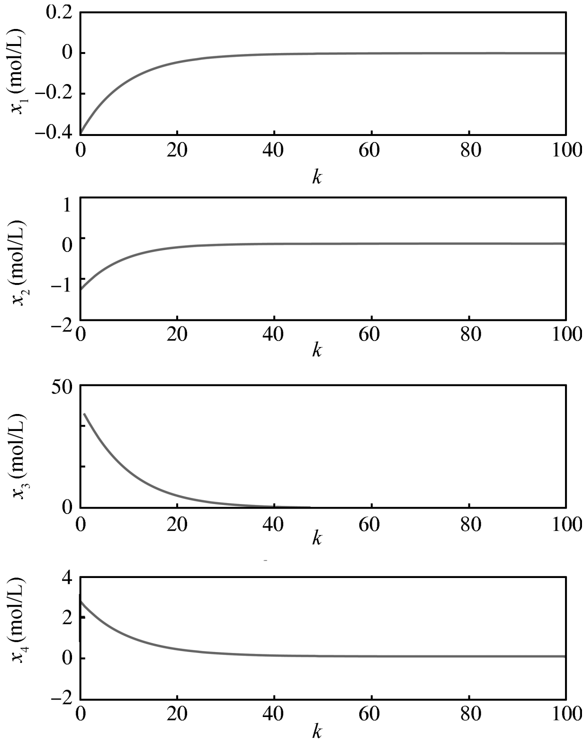

Fig. 1 Simulation results of contractive SM-MPC

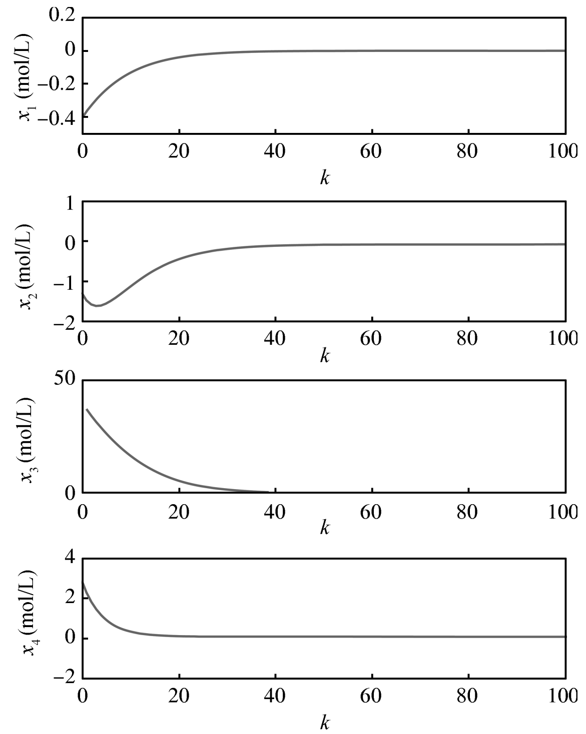

Fig. 2 Simulation results of SM-MPC in [16]

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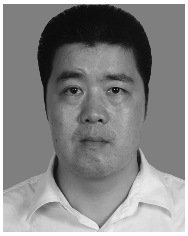

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