

ERRATUM TO: HOLONOMIC D-MODULES ON ABELIAN VARIETIES

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Thomas Krämer and Claude Sabbah pointed out to me that the published proof of Lemma 20.2 only works in the *regular* case. The purpose of this note is to give a correct proof for the general case. Here is the statement again.

Lemma. — *Let $f: A \rightarrow B$ be a surjective morphism of abelian varieties, with connected fibers. If \mathcal{N} is a nontrivial simple holonomic \mathcal{D}_B -module, then $f^*\mathcal{N}$ is a simple holonomic \mathcal{D}_A -module.*

Proof. — Since f is smooth, $f^*\mathcal{N} = \mathcal{O}_A \otimes_{f^{-1}\mathcal{O}_B} f^{-1}\mathcal{N}$ is a holonomic \mathcal{D}_A -module, and so there is a surjective morphism $f^*\mathcal{N} \rightarrow \mathcal{M}$ to a nontrivial simple holonomic \mathcal{D}_A -module \mathcal{M} . We will prove the assertion by showing that it is an isomorphism.

The support $X = \text{Supp } \mathcal{N}$ is an irreducible subvariety of B . As \mathcal{N} is holonomic, there is a dense Zariski-open subset $U \subseteq B$ such that $X \cap U$ is nonsingular and such that the restriction $\mathcal{N}|_U$ is the direct image of a holomorphic vector bundle with integrable connection (\mathcal{E}, ∇) on $X \cap U$ [HTT08, Proposition 3.1.6]. This means that $f^*\mathcal{N}$ is supported on $f^{-1}(X)$, and that its restriction to $f^{-1}(U)$ is the direct image of $(f^*\mathcal{E}, f^*\nabla)$. We observe that, on the fibers of f over points of $X \cap U$, the latter is a trivial bundle of rank $n = \text{rk } \mathcal{E}$.

Since \mathcal{M} is a quotient of $f^*\mathcal{N}$, its restriction to $f^{-1}(U)$ is also the direct image of a holomorphic vector bundle with integrable connection on $f^{-1}(X \cap U)$; as a quotient of $(f^*\mathcal{E}, f^*\nabla)$, the restriction of this bundle to the fibers of f must be trivial of some rank $k \leq n$. Now let $r = \dim A - \dim B$ be the relative dimension of f . By adjunction [HTT08, Corollary 3.2.15], the surjective morphism

$$f^*\mathcal{N} \rightarrow \mathcal{M}$$

gives rise to a nontrivial morphism

$$\mathcal{N} \rightarrow \mathcal{H}^{-r}f_+\mathcal{M}$$

which must be injective because \mathcal{N} is simple. Over U , the left-hand side is a vector bundle of rank n and the right-hand side a vector bundle of rank $k \leq n$; this is only possible if

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$k = n$. But then $f^*\mathcal{N} \rightarrow \mathcal{M}$ is an isomorphism over $f^{-1}(U)$, and since \mathcal{M} is simple, we obtain a short exact sequence

$$0 \rightarrow \mathcal{K} \rightarrow f^*\mathcal{N} \rightarrow \mathcal{M} \rightarrow 0$$

where \mathcal{K} is a holonomic \mathcal{D}_A -module whose support is contained in $f^{-1}(X \setminus X \cap U)$.

Now recall that $f^+\mathcal{N} = f^*\mathcal{N}[r]$. By adjunction [HTT08, Corollary 3.2.15], the inclusion $\mathcal{K} \rightarrow f^*\mathcal{N}$ determines a morphism $f_+\mathcal{K}[r] \rightarrow \mathcal{N}$, which factors as

$$f_+\mathcal{K}[r] \rightarrow \mathcal{H}^r f_+\mathcal{K} \rightarrow \mathcal{N}.$$

Since the support of $\mathcal{H}^r f_+\mathcal{K}$ is contained in $X \setminus X \cap U$, the morphism in question must be zero, and so $\mathcal{K} = 0$. We conclude that $f^*\mathcal{N}$ is isomorphic to \mathcal{M} . \square

REFERENCES

- [HTT08] R. Hotta, K. Takeuchi and T. Tanisaki, *D-Modules, Perverse Sheaves, and Representation Theory*, Progress in Mathematics, vol. 236, Birkhäuser Boston Inc., Boston, 2008.

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