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Book Review

"Physics and Mathematical Tools: Methods and Examples", Alastuey A., Clusel M., Magro M., and Pujol P., World Scientific Publishing, 2016; ISBN: 9789814713245 (softcover)

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The essence of theoretical physics is the conceptualization of physical phenomena in terms of models, which are then investigated by the various mathematical tools. However, the mathematical field and methods are developed so much that even a professional mathematician has problems in embracing the richness of the whole branch. For the physicists, naturalists, and engineers, the knowledge of mathematics and its methods is fundamental, but they cannot know all of it. Therefore, the book, which is intended to present in an accessible, user-friendly way the fundamentals of a several of different fields of mathematics and give the results in a transparent, understandable and useful form, is very desirable.

The goal of this book is to survey selected topics in applied mathematics, including linear response theory, Green's functions method, and saddle point technique, to analyze various physical problems. The book consists of four chapters and 11 appendices. The following pedagogical method is used throughout this book: each chapter contains the preliminaries, including the terse objectives of the chapter, the applications and illustrative examples, and the valuable exercises together with the hints or short solutions to problems, at the end of the book (Appendix J).

Chapter 1, *Linear response and analyticity*, focuses on the basic aspects of the theory of linear response to perturbations of the equilibrium state. The concept of the response function and the general properties of response to a small excitation are derived from the triplet "linearity–causality–stationarity". The idea of susceptibility is introduced, and the Kramers–Kronig relation between the real and imaginary parts of this susceptibility is deduced. These notions and formulas are then generalized, to describe the response of the system to the inhomogeneous perturbations. Then, several examples are presented regarding the miscellaneous aspects of the physics of the linear response including, to mention a few only, an RLC circuit, a dielectric medium, laminar, oscillating flow in a capillary, one-component plasma model (jellium), and the electrical conductivity, for which various derivations of classical and quantum Kubo formulas are presented. The chapter ends with a list of nine exercises. The problems include such topics as computations of response functions and susceptibilities, application of Kramers–Kronig relations for the interstellar medium, and a metal, wave propagation in dielectric media in the context of causality principle, etc. To understand the material, the basic knowledge on complex analysis should be sufficient.

Two chapters refer to the Green's function formalism, one of the most efficient methods in the area of differential equations. Chapter 2, *Static Green's functions*, deals with standard procedures traditionally employed in the construction of Green's functions for boundary-value problems for linear operator equations. The basic properties of Green's functions, such as linearity, superposition, and adjoints, with somewhat more attention devoted to the spectral representation, together with the generalization of the concept of Green's function, are discussed in some detail. The next two sections

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concern the Poisson equation and Helmholtz equation, in three dimensions. The constructions of Green's functions for these equations are presented for various boundary-value conditions. The methods that are used for the construction of Green's functions for the Laplace and Helmholtz operator in low dimensions are described also: in particular, the conformal mapping. Finally, the inhomogeneous partial differential equations, which find their application in plasma physics, or in quantum mechanics, are discussed briefly. The examples begin by considering the origin of the method of images. Next, a ball with uniform motion immersed in a perfect incompressible fluid is analyzed, which illustrates the Green's function approach to the problem with boundary conditions at infinity. The Green's function formalism is explained also through its application to the density of states of a quantum particle and for the scattering particle by a repulsive (Yukawa) potential. In the end, the simple model for the wind blowing on a wall, based on conformal transformations, is formulated and analyzed. Nineteen exercises concentrate on the following issues: Green's functions for the Laplacian in various dimensions; the selected properties of Laplacian Green's functions; the Green's functions relating to electrostatic problems, and regarding the continuum mechanics issues, as well as, the general relativity and the quantum oscillator theory.

Chapter 3, *Dynamical Green's functions*, concerns the dynamical properties of a system, governed by linear partial differential equations containing both space and time variables. The basic definition and some useful properties of time-dependent Green's functions are stated first. Based on the groundwork of Chaps. 1 and 2, the essential details needed to construct the solutions of selected, physically important equations are presented and discussed. These include: the diffusion equation, the non-relativistic Schrödinger equation, the Bloch equation (it describes, at the quantum level, the coupling between light and matter), and the wave equation.

The chapter may seem somewhat difficult, so, as representative examples, the following problems should shed some light. After discussing the spectral construction of causal Green's function for the one-dimensional diffusion problem, the Green's function technique is used for the following issues: the Fraunhofer diffraction, the sound propagation in a perfect fluid, the wavefront structure in supersonic regime, the Cattaneo's equation (the heat equation with the effects of finite speed propagation), and the polarization of the hydrogen atom. Twelve exercises of theoretical and computational interest for readers are formulated, concerning the diffusion and wave equations, the Kelvin equation for long cables, the Cattaneo's equation, and the Klein–Gordon equation, as well as the discussion of the various boundary conditions within the perspective of the partial differential equations.

One of the most important and most powerful methods in asymptotics is the *Saddle point method*. Chapter 4 gives the concise overview of basic methods that can be used for obtaining asymptotic expansions of integrals: the saddle point method and the related approaches: the method of stationary phase and the steepest descent method. The several remarks of the saddle point method, intended to illustrate the principles of the method with simple integral examples, are presented first. Next, the saddle point formula is derived, and the asymptotic nature of this formula is examined, together with the discussion of errors inherent in the method. The classical stationary phase method for one-dimensional integrals is described also. In turn, a brief overview of the saddle point calculations in the complex plane is presented. Two generalizations of the saddle point method are discussed: the case of multiple integral and the case of functional integral. Some subtle remarks about the construction of the functional integral are presented, and the corresponding saddle point formulas are obtained. The next section is almost entirely dedicated to the application of the saddle point method, in statistical mechanics. It begins with deriving and analyzing some useful properties of Stirling's formula. Then, the question of equivalence of canonical and microcanonical ensembles is shortly exposed, and the saddle point approximation method is used to find the asymptotic behavior of suitable integrals. The remaining three examples deal with the stability of a harmonic crystal with respect to thermal fluctuations, the Ising model treated by the functional integral approach and the tunneling of a particle through a repulsive potential barrier, in terms of path integral approximation. Nine problems have been chosen as exercises. Two problems concern the asymptotic behavior of Bessel and Helmholtz Green's functions. Further exercises include the asymptotic evaluation of a wave packet, the Green's function for diffusion operator in context of Cattaneo equation, Ising model with long-range interactions, and the quantum harmonic oscillator. The rest of the exercises cover the calculation of the binomial coefficients, the isothermal–isobaric partition function, and the Bernoulli random trajectories.

The Appendices A–K contain some complementary materials and mathematical notions, as well as the important computational subtleties. Appendix A reviews some elements of the theory of complex functions. A terse presentation of the Laplace transform is given in Appendix B. A method for computing the causal Green's function connected with a one-variable differential operator is the theme of Appendix C. The concept of Hilbert spaces together with the Dirac notation is addressed in Appendix D. Appendix E is devoted to the computational aspects of Gaussian integrals. The basic philosophy of coordinate transformations is outlined in Appendix F, whereas, in Appendix G, the spherical harmonics are defined and elucidated. The important notion of the functional derivative is introduced and discussed in Appendix H. The computed Green's functions for the Laplacian, Helmholtz, and D'Alambert operators, as well as, the diffusion and quantum particle operators, are collected in Appendix J. The last Appendix K plays a role of a useful guide to the mathematical and physical literature related to the issues presented in the book.

This book is fairly clear and comprehensible, making it quite readable. An average math background is necessary to follow the exercises, including some understanding of complex analysis, calculus, linear algebra, and theory of differential equations. The book will be appropriate for use in advanced undergraduate and graduate applied mathematics courses. I recommend it also for the mathematically oriented geophysicists.