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A Proposed Two-Semester Program for Mathematics in the Architecture Curriculum

Proposing a one-year mathematics course for architecture students, the aim of this work is to examine the relevance of mathematics in contemporary architecture, namely its most representative forms of cultural or sport buildings. Because today the architectural object is characterized by a great exuberance, some notions of topology are required; classic linear algebra and analytical geometry are becoming inadequate for the purpose. For the education of an architect, with a modern vision of the utility of technology, the academic staff must understand what students lack, and promote quality in their professional work.

Introduction

All languages (the literary, the artistic, the scientific) evolve almost simultaneously through time. Although it is not clear that there exists a correspondence between elements of architecture and elements of other kinds of language, we may observe that in cultural history these values are unified to express the thematic contexts of each epoch. Examples of this are the relationship between new architecture and new science[1] (see [Jencks 1997]); between theoretical architecture and the theory of Deleuze's fold (see [Perrella 1998]); and between elements of architecture and Mandelbrot's fractals (see [Salat 1992]). We do not intend to explain any of these relationships philosophically. We intend to be objective and practical, to explain how nonlinear analysis may be used at an elementary level to wake up students' minds to new forms that are actually appearing in architecture all over the world. Consequently, to develop their *rationality of feeling* (see [Best 1992]), and later, their professional character.

Traditionally, architectonic forms were constituted by Euclidean objects, namely, the parallelepiped, the sphere, the pyramid, the cone and the cylinder. More recently, new shapes have appeared in architecture [Consiglieri 1994, Consiglieri 2000]. Examples include Frank Gehry's Guggenheim Museum in Bilbao; Peter Eisenman's Aronoff Center in Cincinnati; and Daniel Libeskind's Jewish Extension to the Berlin Museum.

Undeniably, nonlinear forms have also appeared in mathematics. In 1760, Lagrange introduced minimum area surfaces. In 1865, Schwarz formulated problems of minimization and began the thorough study of complex variable functions for their resolution.

In general, most architects want to remain up-to-date and therefore strive to execute these new forms. However, building materials can differ depending on whether the architects find themselves in countries that are developed or not. In developed countries, it may be possible to build with steel, while in less wealthy countries only the use of concrete is feasible. Indeed, several works of Niemeyer in Brazil are indicative of a low gross national product.

In recent architecture, functions are explicitly shown in order that roof framework and points of instability that are extremely important to engineers can be described in wire mesh (see for instance [Adriaenssens, Barnes, and Williams 1999]).

Here, let us concentrate only on forms, i.e., three-dimensional volumes, two-dimensional surfaces or one-dimensional lines, that compose the architectonic object, excluding the details of the structure, such as, for example, the effect of scales on the surface of the Guggenheim Museum in Bilbao, Spain.

Traditionally, geometry is introduced through linear algebra [Alsina and Trillas 1984; Caliò and Marchetti 2000], proceeding to an adequate study of quadratics, conics, rhythms and symmetries. Some authors (see, for instance, [Rego 1992]) hint that topology should be introduced even in the secondary school programme. But other recent books aim introduce the calculus as soon as possible; to present calculus in an intuitive yet intellectually satisfying way; and to illustrate the many applications of calculus to the biological, social and management sciences [Goldstein, Lay, and Schneider 1999; Hille and Salas 1995].

In accordance with this new approach, we propose as an alternative a curriculum of infinitesimal analysis constituted by a programme that is simultaneously theoretical and related to practical applications, as it has been proposed by mathematicians such as Richard Courant [Courant and John 1965], Bento Jesus Caraça [1975] and Elon Lages Lima [2001]. We emphasize the study of functions and their graphical representations, keeping in mind their topological properties and relations, in order to point out the correlation with architectonic forms, and to provide a methodological-cultural background.

Mathematical Analysis for Future Architects

Poincaré did not accept the Platonic concept of mathematical practice as a discovery, but rather he considered mathematical creation a construction, the same as artistic creation, without, however, mixing the two. He called this construction *invention* and he defined it as the capability of building combinations between the combinations already known, not in an arbitrary and useless way but by discerning and choosing [Salazar 1961]. Indeed, an object of art is not the result of science. The object of art appears as a product of imagination or in relationship with the environment (through vision) and reason. Only later come the theoreticians who interpreted the object of art as science presumptions.

Artists, and in particular architects, might not have a scientific thought, but they can be aware of a historical-philosophical overview of science, and consider the engagement of scientists and the present role of the media in divulging science to the society.

Given this state of mind, the important thing is to teach architectural students how to think in order to improve their future flexibility. However, mathematics does not lead to emotional forms but abstract ones; that responsibility belongs to aesthetics.

Alsina and Trilles [1984] have written a thoughtfully elaborated book that introduce some mathematical elements in a clear and easy way for architecture courses. For example, they include: graph theory, linear, affine, Euclidean, equiform and projective geometry, as well as a study of isometries (translations, rotations, symmetries and possible combinations), and compositions of homotheities with isometries. The main inconvenience of the programme given in this book is the time required to classify the figures. Chapters 3, 4 and 5 (linear and affine geometry, determinants and diagonalization, and Euclidean geometry) would fill a whole semester, and for us it is questionable whether we should fill a whole semester with abstract theory and leave aside the rest of the book. The interesting part of applications, such as the theory of symmetry and the conics classification, among others, would either be rigorously presented but incomplete, or complete but lacking in rigour. Moreover, after the initial effort of the first semester concentrated on theory, utterly unintelligible and discouraging for our students, the results would be disappointing. Indeed, a profound learning of the classification of forms is a pedagogic work that unfortunately does not stir the students' imaginations. Consequently, mathematics is disappearing from architecture faculties and colleges.

The study of topology could be an alternative, since topology is more general than geometry and allows the study of the transformations of an object. However, a dense study of the theory in abstract topological spaces is not necessary. Besides, the theory of structures requires the knowledge of differential equations [Heyman 1995]. Therefore, it is sufficient to introduce elementary notions of topology in real space (dimensions 1, 2 and 3). On the other hand, the challenge presented by great architectural surfaces in such public and private institutions as museums, shopping centres, metro or railway stations, sports pavilions or stadiums, requires an adequate knowledge and a new reading so that the topological features of a curve relevant for its graphical representation can be understood.

This panorama of our cultural transition in architectural imageries demands an urgent review of the scientific programmes of mathematics, keeping in mind that a rigorous background is indispensable as a continuity of the upper secondary school. In order to avoid an architectural project becoming the result of inspiration alone, logical analysis must be the first task. This perspective began with the methodology of architectural design by theoreticians Geoffrey Broadbent [1971] and Christopher Alexander [1964] among others, in three stages: analysis, synthesis and evaluation. This systematic method provides an accurate critique of the conception and building processes, and unites logical analytic judgements and emotional creative intentions. In order to facilitate the memorization of symbols, icons, and linguistic elements adequate for the specific objects, the second task is to structure thought. Finally, the development of the mind is the crucial point for the elaboration of today's architectural forms. The study of volumetric characteristics and plasticity of forms requires some notions of topology as well as notions of continuity and differentiability. Consequently, the study of infinitesimal analysis is fundamental in an architecture course.

It is important that mathematics not appear as a mere tool for calculations to resolve structural problems simply because, thanks to today's technology, we can realize any form we want.

A Proposed Two-Semester Programme

Maintaining in the first semester the usual instruments of mathematics for architecture students, we propose a connection between classic analysis and actual calculus in the second semester. Thus, a one-year programme should contain the following topics:

First Semester:

1. Graph theory
2. Geometrical constructions
3. Theory of proportions

Second Semester:

1. Introduction
   - functions of real variables, in dimensions 1, 2, 3
   - composition of functions
2. Notions of topology
3. Continuity
4. Differentiability

Over the years it has become usual to lead the student directly to the heart of the subject and to prepare him for active application of his knowledge. The subject itself is certainly not new, as the contents have been classic since the eighteenth century; what it is new is the ability to explain and illustrate mathematics by examples and applications.

Calculus takes ideas from elementary mathematics (algebra, geometry, trigonometry) enhanced by the limit process, and extends them to a more general situation. While analytical geometry represents curves by functions, calculus follows the reverse procedure, beginning with a function and representing it by a curve. Thus, the study of conics and the theory of symmetry will follow naturally from the study of functions of several variables.

The topics of the second semester must follow the structure:

- conceptualisation: definitions and properties in a n-dimensional space;
- contextualisation: applications of the abstract concepts in real space (n=3);
- manipulation: exercises for concepts training.

This method permits the introduction of various abstract concepts, a comparison between them, and discussion in order to fix them in the students' minds. For this reason, the items must be well balanced, as noted by Elon Lages Lima.[2]

With regards to conceptualisation, it is not fruitful to introduce new abstract mathematical concepts in the faculties of architecture because the teaching of mathematics is almost always neglected. It is preferable to do a thorough study, more rigorous and as often as possible with historical interpretations, of the concepts known since the student's early years, and of how these concepts can be useful for the students in their cultural development.

With regards to contextualisation, the objectives are to exhibit the interaction between mathematical analysis and its various applications, and to emphasize the role of intuition.

The most important aim is to motivate the students. The importance of visualization in developing students' understanding of mathematical concepts is emphasized in the following examples of contextualization. The topics of the second semester curriculum listed above can be related to architectural elements (Figs. 1-6).

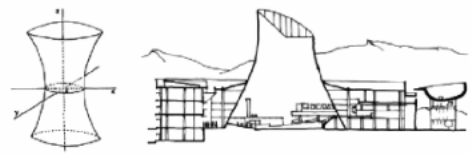

Fig 1. Hyperboloid of one sheet/ Parliament building by Le Corbusier at Chandigarh, India

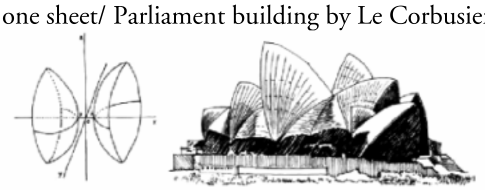

Fig 2. Hyperboloid of two sheets/ Sydney Opera House of Jørn Utzon at Sydney, Australia

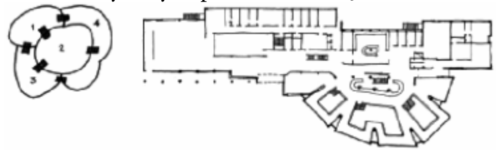

Fig 3. Connected nonconvex set / Rovaniemi Bibliotheca by Alvar Aalto

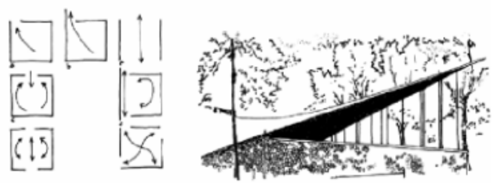

Fig 4. Open and closed sets/ Notre-Dame de la Solitude Chapel by Enrique de la Mora, Mexico

Fig 5. Union of closed sets/ Guggenheim Museum by Frank Lloyd Wright, New York

Fig 6. Topological deformations of surfaces/ Olympic Games Tent by Frei Otto and Gunter Behnisch, Munich. Illustrations are by Teotonio Agostinho

Manipulation

With regards to continuity and differentiability, some examples of manipulation are indispensable for a good understanding of the matter. For instance:

- transformations of graphs: translations, homotheities, symmetries;
- homeomorphisms between lines, surfaces or solids;
- stereographical projections;
- properties of evolutes and involutes;
- circle of curvature as an osculating circle, that is, a curve is osculated (has a contact of order two) at a point P by a circle if they pass through the point P, have the same tangent at P, and also the same curvature when oriented the same way;
- geometrical interpretation of partial derivatives on a saddle-shaped graph;
- constrained optimization and Lagrange multipliers.

Hence, let us suggest the following exercises:

Given a function f of real variable, determine the following graphs: $f(x+c)$, $f(x)+c$, where c is any real constant, $f(ax)$, $af(x)$, for some constant $a>0$, $f(-x)$, $-f(x)$, $|f(x)|$. Compare your graphs with the graph of f, and describe the effect that varying c and a has on the graph of f.

Given the graph of a function f, determine the intervals of the domain in which f is continuous and differentiable. For each point of discontinuity, state whether the discontinuity is a removable discontinuity, a jump discontinuity or neither.

Let $f(x)=\begin{cases} x\sin(1/x), & x\neq 0 \\ 0, & x=0 \end{cases}$ and $g(x)=\begin{cases} x^2\sin(1/x), & x\neq 0 \\ 0, & x=0 \end{cases}$

- Show that f and g are both continuous at 0, and g is differentiable at 0 but f is not differentiable at 0.
- Sketch figures displaying the general nature of the graphs of f and g.

Given a real function f of real variable, build the surface of revolution obtained by rotating f around the *x*-axis, such as a catenary and a catenoid.

Thus, the study of real analysis in various dimensions is a necessary and sufficient condition for the realization of the mentioned requisites, because it contains all the ingredients, namely visualization, intuition, easy understanding, generalization and abstraction, calculus and applications.

Conclusion

Nowadays students do not take any delight in mathematics. A great number scorn it or even hate it. To be conscious of this problem is the first step. The organization of a university-level mathematics module must take into consideration three essential requisites: conceptualisation, contextualisation and manipulation. This process should be measured by both incentive and simplicity of terms.

Unfortunately, the educational laws now in effect in Portugal that govern the first nine years of school (the elementary and lower secondary levels) are such that they allow students not to develop their power of reason. We believe that if the students' interest is awakened, then their reasoning will flourish. So it is essential to bring forward a deep and solid theoretical knowledge simultaneously with the capacity of utilization and application of mathematics. Then students could understand mathematics, perceive its utility, take pleasure in it, and develop their capabilities.

Acknowledgment. Figs. 1-6 are by Teotonio Agostinho

Notes

1. Here the concept Science is understood as the scientific dissemination through the media.
2. Lecture, 'Ensino e Comunicação da Matemática', Fundação Gulbenkian, Lisbon, November 22, 2001.

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