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Preface

This special issue collects six papers on oriented matroids that (except for the first one) were written at the Institut Mittag-Leffler in Djursholm (Sweden), during its program in "Combinatorics" (1991/92) directed by Anders Björner.

Oriented matroids, a concept that was created 20 years ago, has since then grown into a powerful theory. Oriented matroids form a combinatorial model for point configurations and for real hyperplane arrangements (which are *realizable* oriented matroids), but they contain more general objects; the precise geometric meaning of general oriented matroids is given by the "Topological Representation Theorem" of Lawrence, which identifies oriented matroids with arrangements of possibly deformed hyperplanes (known as "pseudoarrangements").

All six papers are concerned with geometric and topological properties of oriented matroids, and in particular with the "new effects" that appear if the condition of realizability is relaxed or dropped. It seems that some of the most interesting phenomena occur at the boundaries of realizability.

The first paper shows how the Topological Representation Theorem can be used to represent an oriented matroid and its dual simultaneously in a single pseudoarrangement.

The second paper, "Combinatorial models for the finite-dimensional Grassmannians," presents a common framework for several situations where spaces of oriented matroids are dealt with. We consider the space of all extensions of a fixed pseudoarrangement by a lower-dimensional pseudosphere (for instance, the space of all pseudolines that can be inserted into an arrangement of hyperplanes). It is a basic conjecture that this space should be a good model for a real Grassmannian if the underlying pseudoarrangement is realizable.

In "Two constructions of oriented matroids with disconnected extension space" the basic conjecture is shown to fail if the realizability assumption is dropped for the oriented matroid used as an ambient space. In "Oriented matroids with few mutations" even more drastic nonlinear effects in oriented matroids are constructed, which—for example—disprove an 18-year-old conjecture of Las Vergnas. Moreover, the constructions presented there yield oriented matroids with considerably fewer simplicial regions than hyperplanes, an effect which arises only in the nonrealizable case.

There are other geometric applications of this line of investigation. Thus, in

"Combinatorial obstructions to the lifting of weaving diagrams" oriented matroids are used to model the "cycles" in incorrect planar drawings of spatial configurations of lines.

Finally, in "What is a complex matroid?" we propose a combinatorial model for the study of discrete structures in *complex space*: we show how the corresponding theory can in part be reduced to oriented matroid theory.

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