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HEAT EXCHANGE IN GAS FURNACES

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The article presents a new method of calculating heat exchange on the heating surfaces in furnaces with a real medium but without scatter, with multiple reflections making a considerable contribution to the heat flow.

1. Statement of the Problem. In calculations of heat exchange the single-zone approximation is widely used: the medium is represented by one zone, and its spectrum is described by integral characteristics. Figure 1 shows the model of a furnace with convex heating surface. Together with the lining and the volume of the furnace, we obtain three isothermal zones. The surfaces are taken to be gray. If the role of the multiple reflections is slight, the calculation may be carried out with the formulas of [1] and of earlier editions beginning in 1934. In gas furnaces the effective optical thickness of the volume may be small, and in the spectrum of the medium, with small dust concentration, "windows" with high conductance of radiant fluxes are obtained. When the surfaces have fairly large reflection coefficients, multiple reflections are obtained which make a considerable contribution to the heat flows. The object of the present work is to take multiple reflections of radiant fluxes with a real spectrum of the medium into account. It is assumed that soot and dust particles are suspended in the gas; these particles do not cause substantial energy dissipation.

A correct solution of the problem with a black heating surface was obtained in [2] but the solution with a reflecting surface was written intuitively. The present author did not succeed in writing the correct solution of the problem with a reflecting heating surface either, but he substantiates a simple roundabout way of solving the problem that is acceptable for practical calculations.

It is expedient to assume that the heat losses through the lining and the convective flows of heat to it are equal. Then the temperature of the lining surface is obtained incidentally in the process of solving the problem. In the given problem of external heat exchange, the temperature of the heating surface is taken to be known. The convective flow on the heating surface is calculated separately, and the density of the resulting radiant flux is determined by the formula

$$q_{p0} = a_T(\theta - \theta_0),\ \mathrm{W/m^2}\ , \tag{1}$$

or more accurately,

$$q_{p0} = KA_0(\varepsilon_1\theta - a_1\theta_0), \tag{2}$$

where $\theta_0 \equiv \sigma T_0^4$; $\theta_* \equiv \sigma T_*^4$; $\theta \equiv \theta_\Gamma \equiv \sigma T_\Gamma^4$. The temperature of the medium $T = T_\Gamma$ is the effective temperature. Specialized literature deals with its calculation. The coefficients a_T and K depend on the number, the dimensions, optical properties, and the geometry of the bodies taking part in the heat exchange. The problem consists in deriving the formulas of these magnitudes for a model of the furnace shown in Fig. 1.

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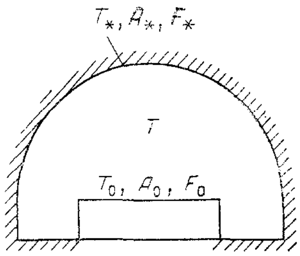

Fig. 1. Model of a reheating furnace.

2. Attempt at Direct Solution. The equations of the full resulting flows on the heating surface and on the lining (Q_{p0} and Q_{p*}, W) have the form

$$Q_{p0} = F_0\theta N_{r0} + F_*A_*\theta_*N_{*0} - F_0A_0\theta_0(N_{0r} + N_{0*}),$$
$$Q_{p*} = F_*\theta N_{r*} + F_0A_0\theta_0N_{0*} - F_*A_*\theta_*(N_{*r} + N_{*0}). \tag{3}$$

In the given sense

$$\sum_k N_{ik} = 1. \tag{4}$$

The coefficients N_{ik} are expressed by infinite series which were substantiated in [2, 3] and other works. For the given model of a furnace we did not manage to find series. If the surfaces were plane and infinite, then according to [3] we would have $N_{0*} = A_*[(1-a_1) + R_0R_*(1-a_3) + R_0^2R_*^2(1-a_5) + \ldots]$. In reality the lining is concave. Between its elements their own multiple reflections occur which combine with the reflections from the heating surface. The series can be written only after various multiple reflections have been grouped. This in particular represents the difficulty that so far has not been overcome. It is assumed that the direct way has to be circumvented. The method consists in solving the problem for the spectra of the medium that are extreme in regard to structure and results: the gray and antigray spectrum. The name of the latter spectrum indicates that in regard to its properties it is the direct opposite of the known gray spectrum. The antigray spectrum consists of lines and bands with rectangular contours, and the spectral absorptivity inside the lines and bands is equal to unity, between them it is equal to zero. The result for any real spectrum lies in the interval between the extreme values obtained for gray and antigray spectra. The final solution therefore includes its interpolation with a view to the extreme results.

3. Solution for the Gray Spectrum. The grouping of bundles and rays and the writing of infinite series is simplified. The reader can fill in the details of the derivation of the series according to the phenomenology: $N_{0*} = A_*'(1 + M' + M'^2 + M'^3 + \cdots) = A_*'/(1-M')$, where $A_*' = A_*\dfrac{1-a_1}{1-\psi}$; $M' = R_0R_*\dfrac{1-\varphi}{1-\psi}(1-\bar{a}_2)$; $1-\bar{a}_1 = (1-\psi)\times(1-a_1)$; $1-\bar{a}_2 = (1-\psi)(1-a_2)/[1-\psi(1-a_1)]$. After substitution we finally obtain

$$N_{0*} = A_*\frac{1-a_1}{1-\psi(1-a_1) - R_0R_*(1-\varphi)(1-a_2)},$$

where $\psi = R_*\varphi$; $\varphi = 1 - \dfrac{F_0}{F_*}$.

Written in abbreviated form, $CN_{0*} = A_*(1-a_1)$, where $C = 1 - R_*\varphi(1-a_1) - R_0R_*(1-\varphi)(1-a_2)$; $CN_{00} = A_0R_*(1-\varphi)$; $CN_{0r} = C[1-(N_{0*}+N_{00})] = a_1[1 + R_*(1-\varphi)(1-a_1)]$.

Although the formulas were derived for gray properties of the medium, we will henceforth use the degrees of blackness and the absorptivities of the volume ε_j and a_j according to a real spectrum. This method of modification of the gray approximation is well known [1]. Since the absorptivity depends on the temperature of the source of the radiant flux, we must add more detailed subscripts to the magnitudes a_1 and a_2 in the formulas we write: $a_1 \to a_{10}$, $a_2 \to a_{20}$. Correspondingly, we must use the magnitudes a_{1*} and a_{2*} in the subsequent group of formulas. Moreover, the geometry of the bodies and the effective thickness of the volume differ on the sides of the surfaces F_0 and F_*. However, later we ignore these and other subtleties of the calculation. In particular, we adopted the following for the sake of simplifying the calculations: $a_{10} = a_{1*} = a_1$, $a_{20} = a_{2*} = a_2$. Then $CN_{*0} = A_0(1-\varphi)(1-a_1)$, $CN_{**} = A_*[\varphi(1-a_1) + R_0(1-\varphi)(1-a_2)]$; $CN_{*\Gamma} = C[1-(N_{*0}+N_{**})] = a_1[1+R_0(1-\varphi)(1-a_1]$.

The values of $N_{\Gamma 0}$ and $N_{\Gamma *}$ are determined from the reciprocal relations $N_{\Gamma 0} = A_0 N_{0\Gamma}$, $N_{\Gamma *} = A_* N_{*\Gamma}$ in which the absorptivities are replaced by the degrees of blackness: $a_1 \to \varepsilon_1$, $a_2 \to \varepsilon_2$.

However, for the sake of simplicity these divergences are also disregarded, with some exceptions in the approximation (2). It is recommended to calculate the coefficient K according to the degree of blackness of the medium averaged over the surface of the shell $F_0 + F_*$. Let us go over to the calculation of the degree of blackness of the furnace space in Eq. (1) without difference between ε_j and a_j with arbitrary number of reflections of the energy fluxes.

The condition $Q_{p*} = 0$ in fact corresponds to the equality of the heat losses through the lining and the convective heat supply to it [1]. Then the second equation (3) enables us to calculate the surface temperature of the lining

$$\theta_*' = \frac{a_1[1+R_0(1-\varphi)(1-a_1)]\,\theta + A_0(1-\varphi)(1-a_1)\,\theta_0}{1-\varphi(1-a_1)-R_0(1-\varphi)(1-a_2)}. \quad (5)$$

The obtained result is useful because direct temperature measurement of the lining entails a large error. This applies all the more to non-steady-state interaction of bodies. An example is a rotary furnace in which the lining over most of the perimeter is in contact with a gas stream. During the time of exposure the temperature rises, and at the speeds of rotation of industrial furnaces its maximum value may be considered to be close to the equilibrium value corresponding to steady-state heat exchange. In that case formula (5) may be used. In the opposite case the temperature is unknown. In mathematical models of heat exchange it is calculated according to rough relationships such as $T_* = (T + T_0)/2$. Our calculations by formula (5) showed that the temperatures T_* and T in rotary furnaces are very close to each other. The difference is of the order of magnitude of 10°.

Substitution of T_* with respect to (5) into the first equation of (3) leads to formula (1), where

$$a_T' = A_0 a_1 K', \quad K' = 1 + \frac{1-a_1+R_0(1-\varphi)(1-a_2)}{1-\varphi(1-a_1)-R_0(1-\varphi)(1-a_2)}. \quad (6)$$

The prime indicates that a_T and K belong to the case with gray properties of the medium.

4. Solution for the Antigray Spectrum. In this case $a_1 = a_2 = a_3 = \ldots$, because the medium does not transmit the proper radiation after its reflection from the surface. Omitting to write the infinite series, we present the final results: $B = 1 - R_*\varphi - R_0 R_*(1-\varphi)$; $BN_{00} = A_0 R_*(1-\varphi)(1-a_1)$; $BN_{**} = A_*(1-a_1)[\varphi + R_0(1-\varphi)]$; $N_{0\Gamma} = N_{*\Gamma} = a_1$; $N_{\Gamma 0} = A_0 a_1$; $N_{\Gamma *} = A_* a_1$.

Substitution and repetition of the transformation of the preceding chapter lead to

$$\theta_*'' \equiv \sigma T_*^4 = \frac{a_1 B\theta + A_0(1-a_1)(1-\varphi)\,\theta_0}{a_1 B + A_0(1-a_1)(1-\varphi)},$$

$$a_T'' = A_0 a_1 K'', \quad K'' = 1 + A_* \frac{1-a_1}{A_* a_1 + A_0(1-\varphi)(1-A_* a_1)}. \quad (7)$$

5. Interpolation of the Extreme Results. According to linear interpolation $\theta_* = \theta_*'' + (\theta_*' - \theta_*'')c$; $T_* = \sqrt[4]{\theta_*/\sigma}$; $K = K'' + (K' - K'')c$, where c is the interpolation coefficient. To determine it, the problem has to be solved in another way. It is simplest to take up the problem of a plane-parallel layer of a real medium bounded by a black cold heating surface and a lining that fully reflects the fluxes. Then

$$a_T' = \varepsilon_1(2-\varepsilon_1), \quad a_T'' = \varepsilon_1, \quad a_T = \varepsilon_2, \quad c = \frac{a_T - a_T''}{a_T' - a_T''} = \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1(1-\varepsilon_1)}.$$

Example. A layer of carbon dioxide with a temperature of 1000°C and an optical thickness of 0.1 m·atm. The degree of blackness is $\varepsilon_1 = 0.1217$; $\varepsilon_2 = 0.1486$. Then we obtain the interpolation coefficient c = 0.252.

TABLE 1. Maximum Temperature of the Lining of a Rotary Furnace with Specified Surface Temperatures of a Layer of the Charge and of the Furnace Medium (°K)

No. of numerical example	T_0	T	T'_*	T''_*	$(T_*)_{max}$
1	1000	1200	1194	1189	1192
2	1600	1800	1794	1789	1792
3	2000	2200	2194	2188	2192

6. Comparison with a Result Known in the Literature. According to the formula of Nevskii and Timofeev [1],

$$\frac{Q_{pT}}{F_1} = \frac{a_1 a_3 (1 - \psi_l a_3)\sigma_0}{a_3 + \psi_l [(1 - a_3)^2 a_1 - a_3^2]} (T_3^4 - T_1^4).$$

With our notation $Q_{pT}/F_1 \equiv q_{p0}$, $a_1 \equiv A_0$, $a_3 \equiv a_1$, $\psi_l = \frac{1-\varphi}{2-\varphi}$, $\sigma_0 T_3^4 \equiv \theta$, $\sigma_0 T_1^4 \equiv \theta_0$ we obtain the formula (1), where

$$a_T = A_0 a_1 \frac{2 - \varphi - (1 - \varphi) a_1}{a_1 (2 - \varphi) + (1 - \varphi)[(1 - a_1)^2 A_0 - a_1^2]}. \quad (8)$$

In deriving this formula, the transmissivities of the volume for the proper and the reflected fluxes of the surface, forming the effective fluxes, did not differ. Strictly, this is possible only with gray spectrum of the medium, when the transformations $(1-a_1)^2 = 1-a_2$, $a_1^2 = 1-2(1-a_1) + (1-a_2)$ are correct; with these, formula (8) is rewritten in the form

$$a_T = A_0 a_1 \left[1 + \frac{1 - a_1 + R_0 (1 - \varphi)(1 - a_2)}{1 - \varphi(1 - a_1) - R_0 (1 - \varphi)(1 - a_2)}\right]. \quad (9)$$

With the appearance of the magnitude a_2, formula (9) expresses the physical phenomena in a system of bodies better than (8). It is easy to see that formulas (9) and (6) coincide.

In case of an antigray spectrum the heat flux is smaller than with a gray spectrum. Correspondingly lower is also the temperature of the lining. Consequently, refining the results in the present work, which is mandatory in order to take multiple reflections correctly into account, leads a priori to lower results than would follow from the previous method of calculation. However, in many problems of practical importance the correction is small.

Let us compare the extreme values of the degree of blackness of a furnace space a'_T and a''_T. With $a_1 = 1$ the formulas coincide. We obtain $a_T = A_0$. This limit corresponds to large furnaces with a dust-containing medium. Then wishing to obtain the maximum divergence, we decrease a_1 and approach the limit $a_1 \to 0$. For $\varphi = 1$ and arbitrary values of the other magnitudes, the results coincide again, $a_T = A_0$. This case corresponds to a rotary furnace containing a small charge. Then we adopt the opposite value $\varphi = 0$, when according to (6)

$$a'_T = A_0 a_1 \frac{2 - a_1}{1 - R_0 (1 - a_2)},$$

and according to (7)

$$a''_T = A_0 a_1 \frac{A_* + A_0 (1 - A_* a_1)}{A_* a_1 + A_0 (1 - A_* a_1)}.$$

This case corresponds to a plane-parallel layer with transmissivity close to unity. In this case in particular, multiple reflections manifest themselves substantially, and the divergence of the results is considerable.

7. Numerical Example. We want to determine the maximum temperature of the lining of a rotary furnace with the following data: $R_0 = 0.3$; $R_* = 0.25$; $a_1 = 0.8$; $a_2 = 0.9$; $\varphi = 0.667$. The degrees of blackness are equal to the absorptivities. The interpolation coefficient is c = 0.7. According to the temperature measurement in the experimental furnace by Karelin et al. [4], the temperature attains its equilibrium value at the end of the contact with the gas phase. We can use the method of the present work consisting in the interpolation of the extreme values of θ'_* and θ''_* obtained on the assumption of a gray and antigray spectrum of

the medium. The calculation for three variants of the specified temperatures T and T_0 is presented in Table 1. It can be seen that with an overall difference of 200°K between temperatures of the gas and of the heating surface, the temperature of the lining is only 8°K lower than the gas temperature. The divergence of the temperatures with gray and antigray spectra of the medium was 5-6°K. The found solution is 2°K lower than the one obtained by the method known in the literature.

NOTATION

a_1, a_2, a_3, absorptivities of the volume relative to the incident flux with black spectrum with single, twofold, threefold, ...passages of the rays through the volume; ε_1, ε_2, ε_3, ..., degrees of blackness of the volume under the same conditions; a_T, degree of blackness of the furnace space; K, a magnitude proportional to it; A, absorptivity (degree of blackness) of the surface; R = 1 − A; B, intermediate magnitude; F, surface area, m^2; N_{ik}, coefficient with the sense of the probability that a quantum of energy emitted by zone i will reach zone k directly or after multiple reflections and will be absorbed in this zone; c, interpolation coefficient; T, temperature, °K; Q_p, power of the resulting heat flux, W; q_p, its density, W/m^2; $\sigma = 5.67 \cdot 10^{-8}$ $W/(m^2 \cdot °K)$; φ, slope from lining to lining regardless of the energy absorption by the medium (approximately). Subscripts: 0, heating surface; *, lining; Γ, volume of the medium.

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SPECTRAL COMPOSITION OF THE EMISSION OF A NONISOTHERMAL STREAM OF COMBUSTION PRODUCTS

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The influence of the temperature and carbon dioxide concentration profiles on the spectral composition of the emission of a stream of combustion products is considered.

The investigation of the temperature fields of gas streams in the working chambers of internally fired furnaces and the fireboxes of boiler plants has great importance for the choice of rational operating and construction parameters assuring the best conditions for heat transfer to the surfaces being heated. The possibilities of determining temperature fields in the volume of a medium on the basis of spectral intensities of incident radiation measured at different wavelengths have been revealed in recent years [1-5].

As is known, the spectral radiation field is the result of the emission and absorption of the medium and the surrounding surfaces and depends on the fields of temperatures and concentration of the emitting components. This results in the possibility of connecting the intensities of spectral emission with the temperature distribution along the viewing line

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