

Estimating Subjective Probabilities: Appendices

by

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Appendix A: Instructions

We provide complete instructions for the introduction to the session (marked I in the top, right corner of the first page of instructions, the quadratic scoring rule task trainer (marked q), the linear scoring rule trainer (marked L), the actual belief elicitation tasks for which subjects are paid (marked sr100), and then the lottery choice tasks (marked LOT). Copies of the exact instructions, which were printed in color, are available on request. Each subject received either the QSR or the LSR instructions.

We employed the following rough timing of each session, to ensure that subjects were finished within the 2-hour time period we had promised when they were recruited:

- 10 minutes demographics
- 15 minutes Eyes Test
- 15 minutes demo of ping pong
- 20 minutes 7 belief tasks including resolving the 3 ping pong cases
- 60 minutes total

- 10 minutes first walk through recording and rolling dice start Hey-Orme
- 10 minutes read Hey-Orme instructions
- 15 minutes Hey-Orme task
- 35 minutes total

- 95 minutes grand total.

YOUR INSTRUCTIONS

I

Welcome! Today we will be asking you several types of questions. Some of these will earn you cash, which we will pay you today. And some may earn you cash which we will pay you in a few weeks. You have already earned \$5 for showing up and agreeing to participate.

There are basically four stages today:

1. We will ask you a series of questions about yourself, such as some basic information about your age. The computer will prompt you for these questions, and you should just work through them at your own pace when we log you in.
2. We will then pause, and provide some instructions on the next task, which involves you making some judgements about what someone in a picture is thinking. We will explain that task when we come to it.
3. We will then pause, and provide more instructions on the next task, which will involve you placing some bets on things that have yet to happen. In this stage we will take small breaks between the bets you place, so that we may explain the next specific thing that you are to bet on. These choices will directly affect your earnings. Nothing comes out of your own pocket.
4. We will then pause, and provide more instructions on some choices you are to make over different amounts of money that have different chances of occurring. These choices will also directly affect your earnings.

The instructions for the second, third and fourth stage will provide more information on the type of choices you are being asked to make.

The experimenters will then collate all of your earnings and pay you for the money you have earned, as well as provide a receipt for any earnings that will be paid in the future.

Your choices are private, and will only be associated with an ID that we will enter when we log you in to the computer. So your name, address and SSN will not be linked to any choices you make. We will pay you privately, one at a time, at the end to keep your earnings private.

Are there any questions? If not, go ahead and answer the questions until the computer pauses and asks for a password. When everyone is finished this stage we will announce the password and we can go on to the second stage. There is no hurry, so take your time.

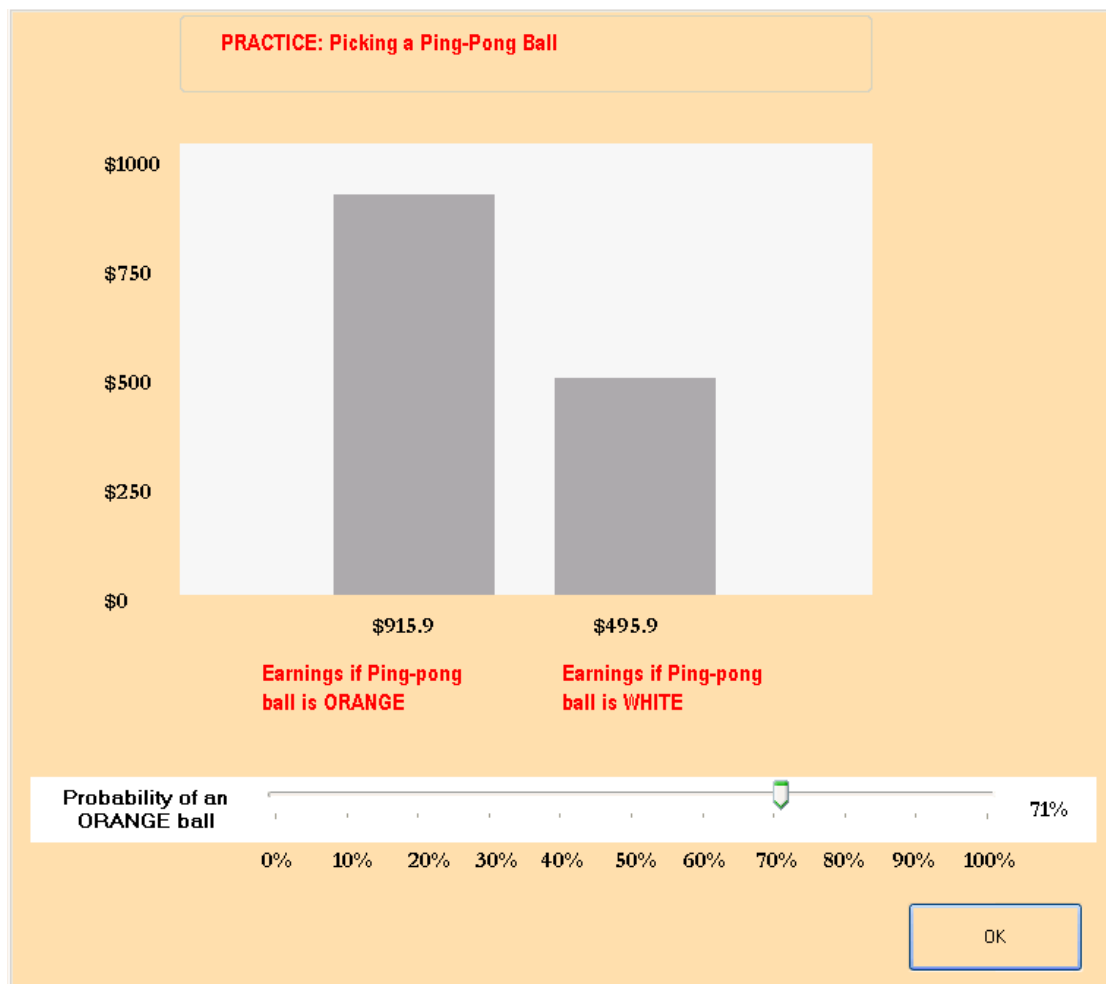
YOU MAY NOW PROCEED WITH THE FIRST STAGE.

INSTRUCTIONS (CONTINUED)

9

In this stage we will give you tasks where you will place bets on the outcome of events that will happen today or in the future. For example, who will be the next U.S. President? You can make more money the more accurately you can predict these outcomes.

You place these bets on a screen like the one below. In a moment we will let you practice with this screen on your computer. Remember, any betting you do today is with our money, not your money.



You place your bets by sliding the bar at the bottom of the screen until you are happy with your choice of a probability report. The computer will start at some point on this bar at random: in the above screen it started at 71%, but you are free to change this as much as you like. In fact, you should slide this bar and see the effects on earnings, until you are happy to confirm your choice.

ENTER THE PASSWORD THAT IS BEING ANNOUNCED NOW.

In this hypothetical example the maximum payoff you can earn is \$1,000. In the actual tasks the maximum payoff will be lower than that, and we will tell you what it is when we come to those

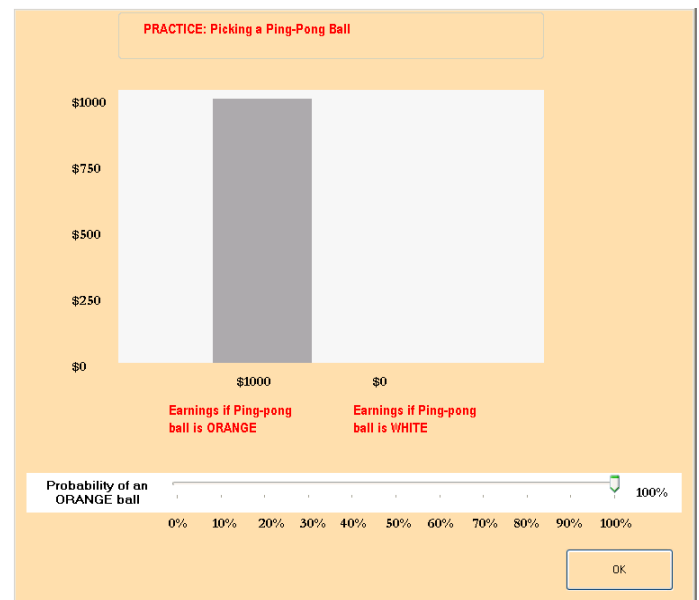
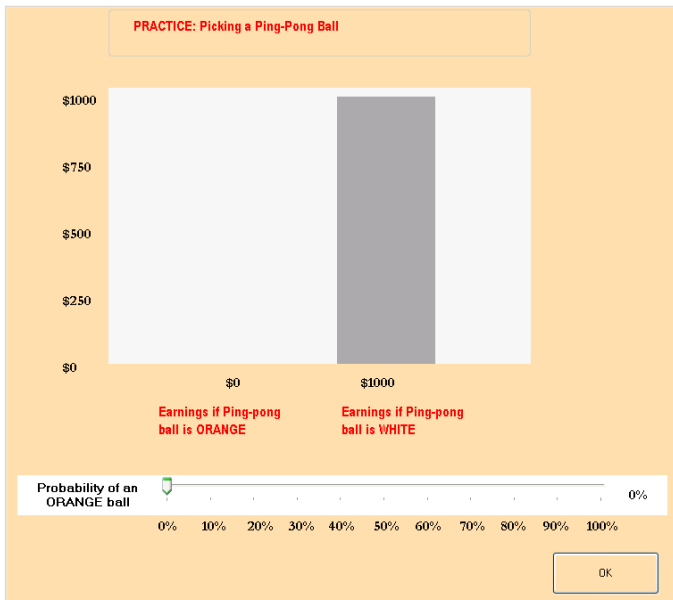
tasks. But the layout of the screen will be the same.

In this demonstration, the event you are betting on is whether a Ping Pong ball drawn from a bingo cage will be Orange or White. We have a bingo cage here, and we have 20 ping-pong balls. 15 of the balls are white, and 5 of the balls are orange. We will give the cage a few turns to scramble them up, and then select one ball at random.

What we want you to do is place a bet on which color will be picked. At the top of your screen we tell you what the event is: in this case, it is **Picking a Ping-Pong Ball**, and you need to bet on whether you think it will be **Orange or White**.

Your possible earnings are displayed in the two bars in the main part of the screen, and also in numbers at the bottom of each bar. For example, if you choose to report 71% you can see that you would earn \$915.90 if the Ping Pong Ball was ORANGE, and \$495.90 if the Ping Pong Ball was WHITE.

Lets see what happens if you make different reports. If you chose to report 0% or 100% here is what you would see, and earn:



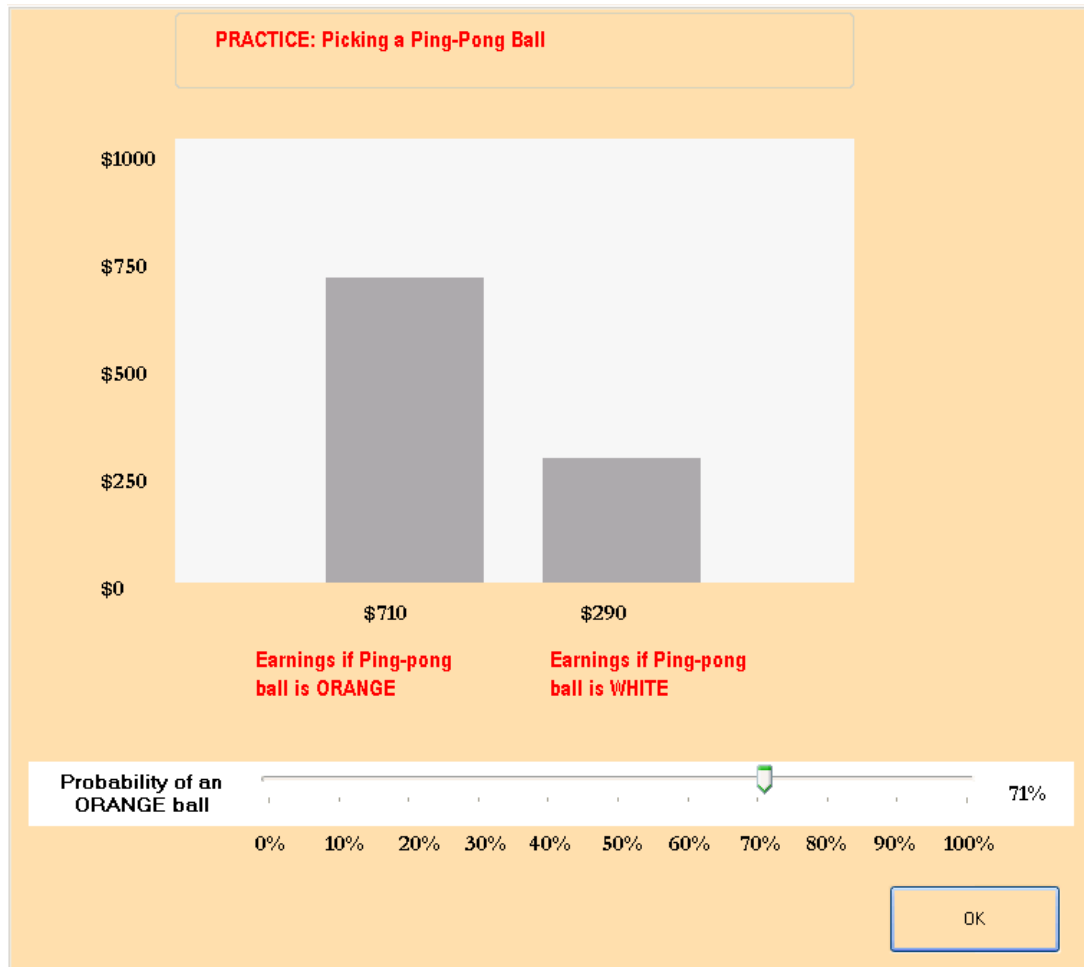
These screens are a little small, but you can see that these two reports lead to extreme payoffs. The “good news” is the possible \$1,000 payoff, but the “bad news” is the possible \$0 payoff. In between the reports of 0% and 100% you will have some positive payoff no matter what happens, but it will vary, as you can see from the report of 71%.

INSTRUCTIONS (CONTINUED)

L

In this stage we will give you tasks where you will place bets on the outcome of events that will happen today or in the future. For example, who will be the next U.S. President? You can make more money the more accurately you can predict these outcomes.

You place these bets on a screen like the one below. In a moment we will let you practice with this screen on your computer. Remember, any betting you do today is with our money, not your money.



You place your bets by sliding the bar at the bottom of the screen until you are happy with your choice of a probability report. The computer will start at some point on this bar at random: in the above screen it started at 71%, but you are free to change this as much as you like. In fact, you should slide this bar and see the effects on earnings, until you are happy to confirm your choice.

ENTER THE PASSWORD THAT IS BEING ANNOUNCED NOW.

In this hypothetical example the maximum payoff you can earn is \$1,000. In the actual tasks

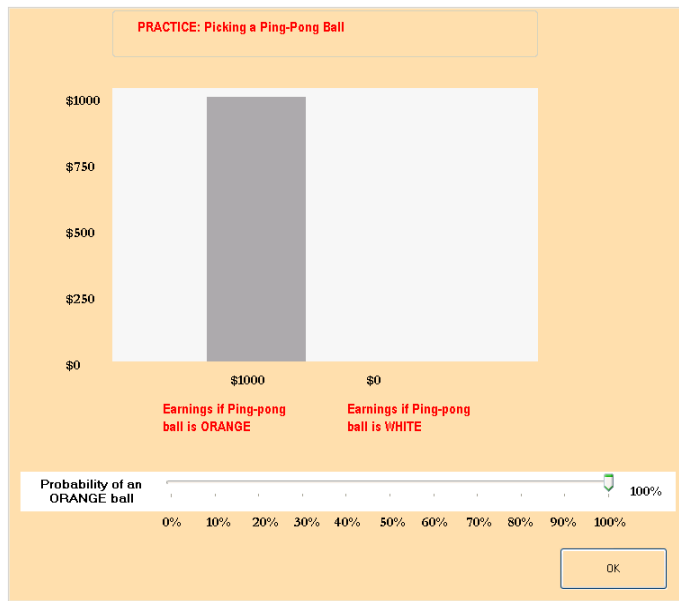
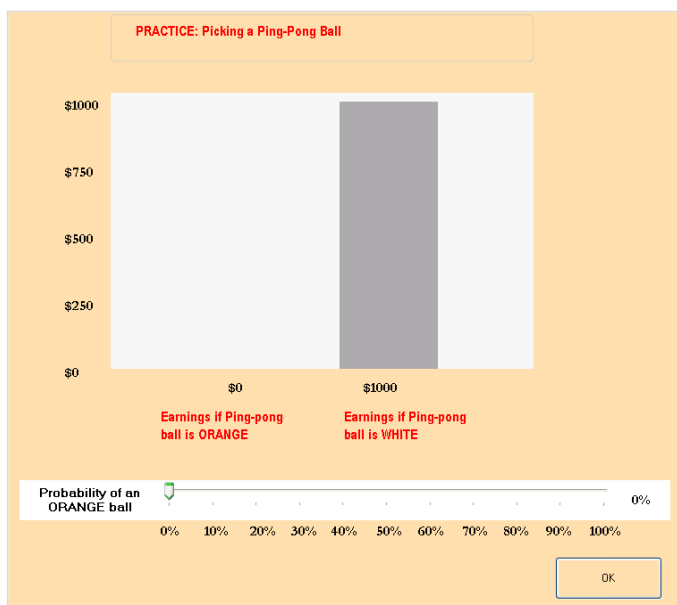
the maximum payoff will be lower than that, and we will tell you what it is when we come to those tasks. But the layout of the screen will be the same.

In this demonstration, the event you are betting on is whether a Ping Pong ball drawn from a bingo cage will be Orange or White. We have a bingo cage here, and we have 20 ping-pong balls. 15 of the balls are white, and 5 of the balls are orange. We will give the cage a few turns to scramble them up, and then select one ball at random.

What we want you to do is place a bet on which color will be picked. At the top of your screen we tell you what the event is: in this case, it is **Picking a Ping-Pong Ball**, and you need to bet on whether you think it will be **Orange or White**.

Your possible earnings are displayed in the two bars in the main part of the screen, and also in numbers at the bottom of each bar. For example, if you choose to report 71% you can see that you would earn \$710 if the Ping Pong Ball was ORANGE, and \$290 if the Ping Pong Ball was WHITE.

Lets see what happens if you make different reports. If you chose to report 0% or 100% here is what you would see, and earn:



These screens are a little small, but you can see that these two reports lead to extreme payoffs. The “good news” is the possible \$1,000 payoff, but the “bad news” is the possible \$0 payoff. In between the reports of 0% and 100% you will have some positive payoff no matter what happens, but it will vary, as you can see from the report of 71%.

Summarizing, then, there are two important points for you to keep in mind when placing your bets:

1. **Your belief about the chances of each outcome is a personal judgment that depends**

on information you have about the different events. In this practice example, the information you have consists of the total number of Orange balls and White balls.

2. **Your choices might also depend on your willingness to take risks or to gamble.**

There is no right choice for everyone. For example, in a horse race you might want to bet on the longshot since it will bring you more money if it wins. On the other hand, you might want to bet on the favorite since it is more likely to win something.

For each task, your choice will depend on two things: your judgment about how likely it is that each outcome will occur, and how much you like to gamble or take risks.

You will now make your report in this practice round. When you have chosen the report, confirm your bet by clicking on the OK tab.

After you click OK, a special box will come up which causes the program to pause. We will do this after every series of bets, and then explain what the next few bets are about. We will tell you what the password is when we are all ready to proceed. There is plenty of time, so there is no need to rush.

When everyone has placed their bets we will pick the ball and you will see what your earnings would have been if this had been for money. After that we will go on with the bets for which you can earn real money.

Does anyone have any questions?

ENTER THE PASSWORD THAT IS BEING ANNOUNCED NOW.

We are now ready to begin the choices for which you will be paid. There will be several sets of choices. In each case we will describe the bet to you, and then you can place your bets. Then we will explain the next couple of bets to you, and you place those bets.

Some of these bets will be about outcomes we know today, here and now, and some will be about outcomes we will only know in a few weeks. There will be 7 bets in all. We will pay you for one of these 7 bets. We will pick this bet at random after all bets are made, and tell you which one will be paid. You should view each bet as if it could be the one to determine your payoffs, since one of them actually will.

The maximum payoff for your bets today will be \$100.

Ping Pong Balls Again

We will now repeat the task with Ping Pong balls a few times.

We have a number of ping pong balls in each of three bingo cages, which we have labeled Cage A, Cage B and Cage C. Some of the ping pong balls are Orange and some are White. We will roll each bingo cage and you can decide for yourself what fraction of Orange balls you think are in the cage. Of course, the balls will be rolling around, and you may not be able to tell exactly how many Orange balls are in the cage. You will be asked to bet on the color of one ping pong ball, selected at random after you all place your bets. For example, if there are 20 Orange balls and 80 White balls, the chance of an Orange ball being picked at random is $20 \div 100$, or 20%.

We will do this task 3 times, with 3 different bingo cages. Just be sure that you check which cage you are placing a bet on. You can see this listed in the top left corner of your screen, where it refers to Cage A, Cage B or Cage C. We will show you each cage one at a time, and allow you to place your bets after we show it to you.

Do you have any questions?

The Eyes Test

At the beginning of today's experimental session we asked you to answer 36 questions, called **The Eyes Test**. These questions were designed by psychologists to measure a person's ability to "read someone else's mind" by just looking at their eyes.

Each and everyone of you were given the same 36 Eyes Test questions in today's experiment and a total score was recorded for each and every one of you in this experiment.

Now we come to the outcome we want you to place bets on in this portion of the experiment. We will pick one man and one woman in the room. Do you think the man who is selected will have a higher score on the Eyes Test than the woman who is selected?

After everyone in the experiment has made their bets for this event we will randomly select one man, and randomly select one woman from this experimental session. We will use the cards we collected, and sort them into one pile for men and one pile for women. Each pile will be shuffled, and one card drawn from each pile. We will then compare the score for the man that is drawn with the score for the woman that is drawn, and write these scores up on the board for you to see.

We therefore pose the following outcome for you to bet on now:

That the man we select at random will have a higher score on the Eyes Test than the woman we select at random.

Do you have any questions?

2008 Presidential Elections

We want you to place bets on some questions about the U.S. Presidential Elections being held in a few weeks:

5. Will the next President of the United States be a Democrat or a Republican?
6. Will the popular vote for the winning candidate be 5 or more percentage points greater than the popular vote for the losing candidate?
7. Will the popular vote for the winning candidate be 10 or more percentage points greater than the popular vote for the losing candidate?

It is important that you understand that the first question is about the outcome of the Electoral College vote, and not the popular vote. The popular vote is just the sum of all votes across the United States. We are only referring to the Presidential Election, and not to any other elections that might occur on the same day.

For the second and third question, we are asking if you think that the winner of the popular vote will beat the loser by 5 or 10 percentage points or more. For example, if the winner of the popular vote gets 51% of the vote and the loser gets 49%, then this is a 2 percentage point difference. If the winner gets 53% and the loser gets 47%, then this is a 6 percentage point difference.

The election will be on Tuesday, November 4, 2008. To use a widely respected public source for the outcome, we will use the New York Times of Friday, November 7, 2008 as the official source used to determine your payoffs. In the event that there is a drawn out determination of the next President, such as in the 2000 election, we will delay payments until Inauguration Day, which is on January 20, 2009.

You will be paid for your bets in checks that will be mailed out on Monday, November 10, assuming we know who the next President will be at that time.

Please go ahead now and place your bets for this event, unless you have any questions.

INSTRUCTIONS (CONTINUED)

LOT

This is the final stage of today's experiment. You will be asked to choose between lotteries with varying prizes and chances of winning. You will be presented with a series of lotteries where you will make choices between pairs of them. There are 45 pairs in the series. For each pair of lotteries, you should indicate which of the two lotteries you prefer to play. You will actually get the chance to play one of the lotteries you choose, and will be paid according to the outcome of that lottery, so you should think carefully about which lotteries you prefer.

Here is an example of what the computer display of such a pair of lotteries will look like. The display on your screen will be bigger and easier to read.



The outcome of the lotteries will be determined by the draw of a random number between 1 and 100. Each number between (and including) 1 and 100 is equally likely to occur. In fact, you will be able to draw the number yourself using two 10-sided dice.

In the above example the left lottery pays five dollars (\$5) if the number on the dice that is rolled is between 1 and 40, and pays fifteen dollars (\$15) if the number is between 41 and 100. The yellow color in the pie chart corresponds to 40% of the area and illustrates the chances that the

number on the dice rolled will be between 1 and 40 and your prize will be \$5. The black area in the pie chart corresponds to 60% of the area and illustrates the chances that the number on the dice rolled will be between 41 and 100 and your prize will be \$15.

We have selected colors for the pie charts such that a darker color indicates a higher prize. White will be used when the prize is zero dollars (\$0).

Now look at the pie in the chart on the right. It pays five dollars (\$5) if the number on the dice rolled is between 1 and 50, ten dollars (\$10) if the number is between 51 and 90, and fifteen dollars (\$15) if the number is between 91 and 100. As with the lottery on the left, the pie slices represent the fraction of the possible numbers which yield each payoff. For example, the size of the \$15 pie slice is 10% of the total pie.

Each pair of lotteries is shown on a separate screen on the computer. On each screen, you should indicate which of the lotteries you prefer to play by clicking on one of the three boxes beneath the lotteries. You should click the LEFT box if you prefer the lottery on the left, the RIGHT box if you prefer the lottery on the right, and the DON'T CARE box if you do not prefer one or the other.

You should approach each pair of lotteries as if it is the one out of the 45 that you will play out. If you chose DON'T CARE in the lottery pair that we play out, you will pick one using a 10-sided die, where the numbers 1-5 correspond to the left lottery and the numbers 6-10 to the right lottery.

After you have worked through all of the pairs of lotteries, raise your hand and an experimenter will come over. You will then roll two 10-sided die to determine which pair of lotteries that will be played out. You roll the die until a number between 1 and 45 comes up, and that is the lottery pair to be played. If you picked DON'T CARE for that pair, you will use the 10-sided die to decide which one you will play. Finally, you will roll the two 10-sided dice to determine the outcome of the lottery you chose.

For instance, suppose you picked the lottery on the left in the above example. If the random number you rolled was 37, you would win \$5; if it was 93, you would get \$15. If you picked the lottery on the right and drew the number 37, you would get \$5; if it was 93, you would get \$15.

Therefore, your payoff is determined by three things:

- by which lottery pair is chosen to be played out in the series of 45 such pairs using the two 10-sided die;
- by which lottery you selected, the left or the right, for that pair; and
- by the outcome of that lottery when you roll the two 10-sided die.

This is not a test of whether you can pick the best lottery in each pair, because none of the lotteries are necessarily better than the others. Which lotteries you prefer is a matter of personal taste. The people next to you will have different lotteries in front of them when you make your choices, and may have different tastes, so their responses should not matter to you. Please work silently, and make your choices by thinking carefully about each lottery.

All payoffs are in cash, and are in addition to the \$5 show-up fee that you receive just for being here and any earnings from the previous stage.

We will now come around to your computer and get you started. When you are finished, please signal someone to come around to play out your lottery and record your earnings. As soon as you have finished the actual series, and after you have rolled the necessary dice, you will be asked to check with someone in the next room to make sure that your earnings sheet is complete, and then you will be paid. You are then free to go. Thanks again for your participation today!

Appendix B. Additional Statistical Results

Table B1: Estimates of the Utility Function Under EUT

Variable	Description	Estimate	Standard Error	<i>p</i> -value	Lower 95% Confidence Interval	Upper 95% Confidence Interval
A. No Covariates						
r		0.30	0.03	<0.01	0.23	0.36
α		0.03	0.01	<0.01	0.02	0.04
μ		0.07	0.00	<0.01	0.07	0.08
B. Adding Covariates						
r						
Constant		0.23	0.06	<0.01	0.12	0.35
female	Female	0.14	0.07	0.04	0.01	0.28
Over22	Over 22 Years of age	0.09	0.10	0.37	-0.11	0.29
GPAhi	GPA great than 3.75	0.05	0.09	0.57	-0.13	0.23
Graduate	Graduate Student	-0.16	0.12	0.17	-0.40	0.07
α						
Constant		0.02	0.01	0.02	0.00	0.03
female	Female	0.02	0.01	0.18	-0.01	0.05
Over22	Over 22 Years of age	0.01	0.01	0.39	-0.02	0.04
GPAhi	GPA great than 3.75	0.00	0.02	0.96	-0.03	0.03
Graduate	Graduate Student	0.00	0.02	0.95	-0.03	0.03
μ		0.07	0.00	<0.01	0.07	0.08

Table B2: Estimates of the Utility Function Under RDU

Variable	Description	Estimate	Standard Error	<i>p</i> -value	Lower 95% Confidence Interval	Upper 95% Confidence Interval
A. No Covariates						
\hat{r}		0.49	0.05	<0.01	0.39	0.58
$\hat{\alpha}$		0.07	0.02	<0.01	0.03	0.12
$\hat{\gamma}$		0.72	0.06	<0.01	0.60	0.83
$\hat{\mu}$		0.08	0.00	<0.01	0.07	0.09
B. Adding Covariates						
\hat{r}						
Constant		0.45	0.07	<0.01	0.31	0.58
female	Female	0.08	0.10	0.93	-0.18	0.20
Over22	Over 22 Years of age	0.02	0.15	0.87	-0.28	0.33
GPAhi	GPA great than 3.75	0.07	0.10	0.48	-0.12	0.26
Graduate	Graduate Student	0.18	0.25	0.46	-0.30	0.67
$\hat{\alpha}$						
Constant		0.05	0.02	0.04	0.00	0.09
female	Female	-0.01	0.04	0.82	-0.08	0.06
Over22	Over 22 Years of age	0.03	0.08	0.72	-0.12	0.18
GPAhi	GPA great than 3.75	0.07	0.07	0.30	-0.06	0.21
Graduate	Graduate Student	0.29	0.37	0.42	-0.42	1.01
$\hat{\gamma}$						
Constant		0.72	0.07	<0.01	0.59	0.86
female	Female	0.15	0.10	0.15	-0.05	0.35
Over22	Over 22 Years of age	0.05	0.25	0.85	-0.45	0.54
GPAhi	GPA great than 3.75	-0.09	0.13	0.46	-0.34	0.16
Graduate	Graduate Student	-0.47	0.41	0.25	-1.28	0.33
$\hat{\mu}$		0.07	0.01	<0.01	0.06	0.08

Table B3: Detailed EUT Estimates for Bets on Win Margin Being 5% or More

Variable	Description	Estimate	Standard Error	<i>p</i> -value	Lower 95% Confidence Interval	Upper 95% Confidence Interval
\mathbf{r}						
Constant		0.22	0.07	<0.01	0.07	0.33
female	Female	0.16	0.09	0.05	0.00	0.33
Over22	Over 22 Years of age	0.12	0.12	0.32	-0.12	0.36
GPAhi	GPA great than 3.75	0.06	0.10	0.57	-0.14	0.26
Graduate	Graduate Student	-0.19	0.14	0.16	-0.47	0.07
α						
Constant		0.01	0.01	0.14	0.00	0.03
female	Female	0.02	0.02	0.27	-0.01	0.05
Over22	Over 22 Years of age	0.01	0.02	0.61	-0.03	0.03
GPAhi	GPA great than 3.75	0.00	0.02	0.98	-0.03	0.03
Graduate	Graduate Student	0.00	0.02	0.87	-0.04	0.04
μ_{RA}	Fechner Error	0.09	0.01	<0.01	0.07	0.10
π						
Constant		-0.74	0.28	<0.01	-1.30	-0.19
female	Female	0.32	0.24	0.20	-0.17	0.81
Over22	Over 22 Years of age	-0.17	0.39	0.66	-0.93	0.59
GPAhi	GPA great than 3.75	0.16	0.23	0.48	-0.28	0.60
Graduate	Graduate Student	-0.16	0.48	0.73	-1.12	0.78
LSR	Linear Scoring Rule tasks	0.34	0.27	0.21	-0.19	0.86

Figure B1: Alternative Probability Weighting Functions
with Rank Dependent Utility Model of Choices
over Objective Risk

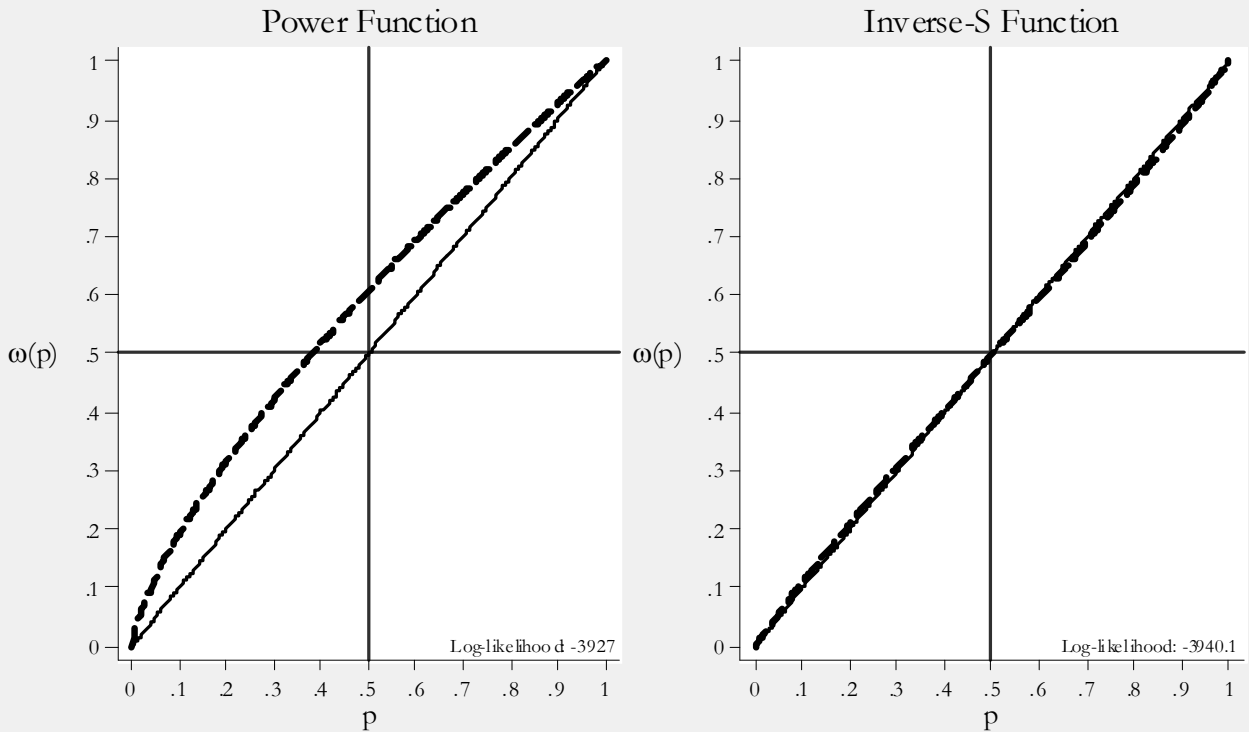
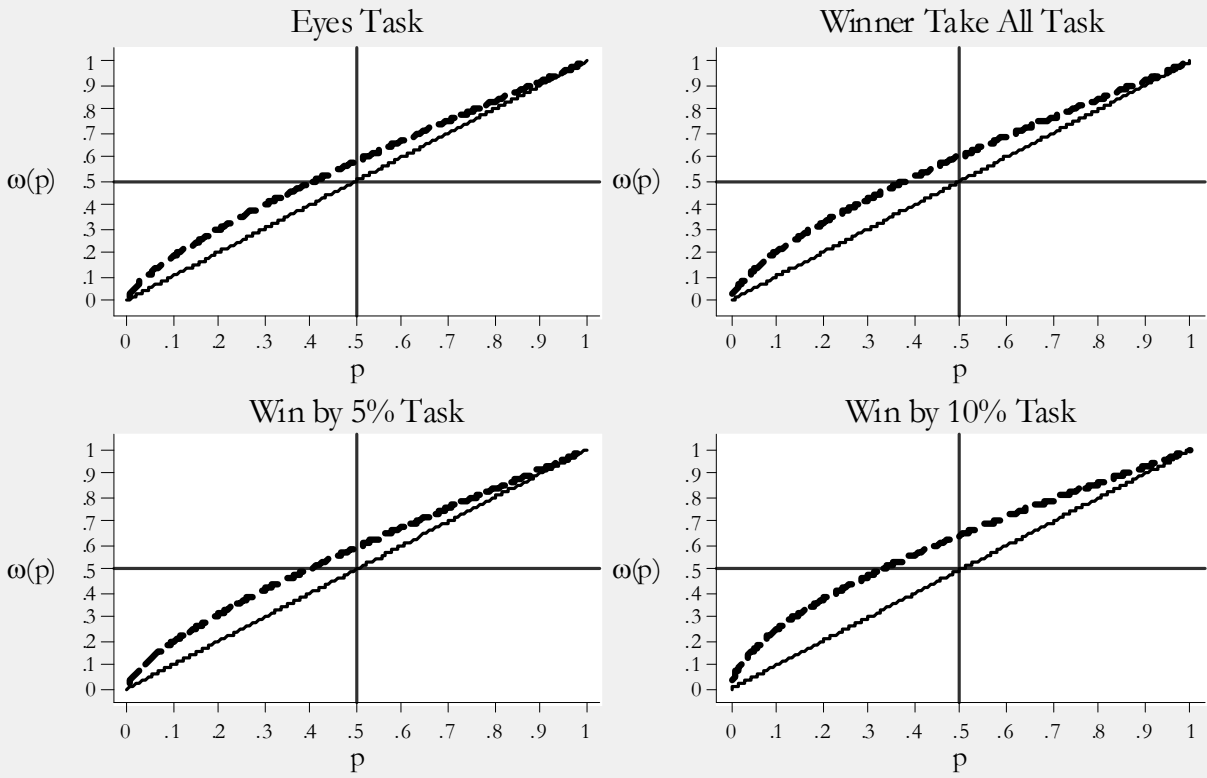


Figure B2: Power-Prelec Probability Weighting Functions
Estimated over Objective and Subjective Risk Tasks



Appendix C: Econometric Specification

A. Risk Attitudes under Expected Utility Theory

We assume an Expo-Power (EP) utility function originally proposed by Saha [1993]. Following Holt and Laury [2002], the EP function is defined as

$$u(y) = [1 - \exp(-\alpha y^{1-r})] / \alpha, \quad (1)$$

where α and r are parameters to be estimated, and y is income from the experimental choice. The EP function can exhibit increasing or decreasing relative risk aversion (RRA), depending on the parameter α : RRA is defined by $r + \alpha(1-r)y^{1-r}$, so RRA varies with income if $\alpha \neq 0$ and the estimate of r defines RRA at a zero income. This function nests CRRA (as $\alpha \rightarrow 0$) and CARA (as $r \rightarrow 0$). The functional form of utility employed here is of no importance, and any monotonic increasing function of $u(\cdot)$ could have been implemented.

The utility function (1) can be estimated using maximum likelihood and a latent EUT structural model of choice. Let there be K possible outcomes in a lottery; in our lottery choice task $K \leq 4$. Under EUT the probabilities for each outcome k in the lottery choice task, p_k , are those that are induced by the experimenter, so expected utility is simply the probability weighted utility of each outcome in each lottery i :

$$EU_i = \sum_{k=1,K} [p_k \times u_k]. \quad (2)$$

The EU for each lottery pair is calculated for a candidate estimate of r and α , and the index

$$\nabla EU = eu_R / (eu_L + eu_R) \quad (3)$$

calculated, where

$$eu_i = \exp(EU_i), \quad (4)$$

for $i = \{R, L\}$, and where EU_L is the “left” lottery and EU_R is the “right” lottery, as displayed to the subject and illustrated in Figure 1. This latent index, based on latent preferences, is already in the form of a probability.¹

Thus the likelihood of the observed responses, conditional on the EUT and EP specifications being true, depends on the estimates of r and α given the above statistical specification

¹ It is well known, but useful to note, that (3) is equivalent to $\Lambda(EU_R - EU_L)$ where $\Lambda(\cdot)$ is the logistic cumulative density function. Thus (3) embodies a statistical “link function” between the difference in the EU of the two lotteries and the probability of the observed choice. Again, the specific “link function” is of no importance here, and we could have implemented any link function mapping the utility difference into a choice probability.

and the observed choices. If we ignore responses that reflect indifference² the log-likelihood is then

$$\ln L(\mathbf{r}, \alpha; \mathbf{y}, \mathbf{X}) = \sum_i [(\ln (\nabla EU) \times \mathbf{I}(y_i = 1)) + (\ln (1 - \nabla EU) \times \mathbf{I}(y_i = -1))] \quad (5)$$

where $\mathbf{I}(\cdot)$ is the indicator function, $y_i = 1(-1)$ denotes the choice of the Option R (L) lottery in risk aversion task i , and \mathbf{X} is a vector of individual characteristics reflecting age, sex, race, and so on. When we pool responses over subjects the \mathbf{X} vector will play an important role to allow for some heterogeneity of preferences.

To allow for observed subject heterogeneity with respect to risk attitudes, the parameters \mathbf{r} and α are each modeled as linear functions of observed individual characteristics of the subject.³ For example, assume that we only had information on the age and sex of the subject, denoted Age (years of age) and Female (0 for males, and 1 for females). Then we would estimate the coefficients r_0 , r_1 and r_2 in $\mathbf{r} = r_0 + r_1 \times \text{Age} + r_2 \times \text{Female}$. Therefore, each subject would have a different estimated \mathbf{r} , $\hat{\mathbf{r}}$, for a given set of estimates of r_0 , r_1 and r_2 to the extent that the subject had distinct individual characteristics. So if there were two subjects with the same sex and age, to use the above example, they would literally have the same $\hat{\mathbf{r}}$, but if they differed in sex and/or age they would generally have distinct $\hat{\mathbf{r}}$. In fact, we use four individual characteristics to model heterogeneity in our estimated risk preferences. Apart from a dummy for people aged over 22 years and gender, these include binary indicators for subjects that self-declare as having a high GPA (over 3.75) and self-declare as graduate students.

An important extension of the core model is to allow for subjects to make some errors. The notion of error is one that has already been encountered in the form of the statistical assumption (4) that the probability of choosing a lottery is not 1 when the EU of that lottery exceeds the EU of the other lottery.⁴ By varying the shape of the link function implicit in (4), one can informally imagine subjects that are more sensitive to a given difference in the index ∇EU and subjects that are not so sensitive. We use the contextual error specification proposed by Wilcox [2011]. It posits the latent index

² In our lottery experiments the subjects are told at the outset that any expression of indifference would mean that the experimenter would toss a fair coin to make the decision for them if that choice was selected to be played out. Hence one can modify the likelihood to take these responses into account either by recognizing this is a third option, the compound lottery of the two lotteries, or alternatively that such choices imply a 50:50 mixture of the likelihood of choosing either lottery, as illustrated by Harrison and Rutström [2008; p.71]. We do not consider indifference here because it was an extremely rare event.

³ Alternatively we could allow for unobserved individual heterogeneity in our maximum likelihood estimation by specifying fixed effects, random effects or random coefficients, and constrain the variance matrix accordingly. Andersen, Fountain, Harrison, Hole and Rutström [2012] illustrate how to apply the random coefficients approach to estimating “imprecise” subjective probabilities.

⁴ This assumption is clear in the use of a link function from the difference between the EU of each option to the probability of picking one or other lottery; in the case of the logistic CDF that is implied by our approach in (3), this link function is $\Lambda(EU_R - EU_L)$. If the subject exhibited no errors from the perspective of EUT, this link function would instead be a step function: zero for all values of $(EU_R - EU_L) < 0$, anywhere between 0 and 1 for $(EU_R - EU_L) = 0$, and 1 for all values of $(EU_R - EU_L) > 0$. Harrison [2008; p.326] illustrates the implied CDF, referring to it as the CDF of a “Hardnose Theorist.”

$$eu_i = \exp[(EU_i/v)/\mu], \quad (4')$$

instead of (4), where v is a normalizing term for each lottery pair L and R, and $\mu > 0$ is a structural “noise parameter” used to allow some errors from the perspective of the deterministic EUT model. The normalizing term v is defined as the maximum utility over all prizes in this lottery pair minus the minimum utility over all prizes in this lottery pair, and ensures that the *normalized* EU difference $[(EU_R - EU_L)/v]$ remains in the unit interval. As $\mu \rightarrow \infty$ this specification collapses ∇EU to 0 for any values of EU_R and EU_L , so the probability of either choice converges to $1/2$. So a larger μ means that the difference in the EU of the two lotteries, conditional on the estimate of r , has less predictive effect on choices. Thus μ can be viewed as a parameter that flattens out, or “sharpens,” the link functions implicit in (4). This is just one of several different types of error story that could be used, and Wilcox [2008] provides a masterful review of the implications of the strengths and weaknesses of the major alternatives.

Thus we extend the likelihood specification to include the noise parameter μ and maximize $\ln L(r, \alpha, \mu; y, \mathbf{X})$ by estimating r , α and μ , given observations on y and \mathbf{X} .⁵ Additional details of the estimation methods used, including corrections for “clustered” errors when we pool choices over subjects and tasks, is provided by Harrison and Rutström [2008; p.69ff].

B. Risk Attitudes under Rank Dependent Utility Theory

The RDU model extends the EUT model by allowing for decision weights on lottery outcomes. Instead of (1) we have

$$u(y) = [1 - \exp(-\acute{\alpha}y^{1-\acute{r}})]/\acute{\alpha}, \quad (6)$$

where \acute{r} and $\acute{\alpha}$ are the coefficient for the curvature of the utility function, comparable to r and α in (1). To calculate decision weights under RDU one replaces expected utility defined by (2) with RDU

$$RDU_i = \sum_{k=1, K} [w_k \times u_k] \quad (7)$$

where

$$w_i = \omega(p_i + \dots + p_n) - \omega(p_{i+1} + \dots + p_n) \quad (8)$$

for $i=1, \dots, n-1$, and

$$w_i = \omega(p_i) \quad (9)$$

for $i=n$, the subscript indicating outcomes ranked from worst to best, and where $\omega(p)$ is some probability weighting function. In general, we adopt the simple “power” probability weighting function proposed by Quiggin [1982], with curvature parameter γ :

⁵ The normalizing term v is given by the value of r and the lottery parameters, which are part of \mathbf{X} .

$$\omega(p) = p^\gamma \tag{10}$$

So $\gamma \neq 1$ is consistent with a deviation from the conventional EUT representation.

An alternative probability weighting function is the “inverse-S” function popularized by Tversky and Kahneman [1992]:

$$\omega(p) = p^\gamma / (p^\gamma + (1-p)^\gamma)^{1/\gamma} \tag{11}$$

This function exhibits inverse-S probability weighting (optimism for small p , and pessimism for large p) for $\gamma < 1$, and S-shaped probability weighting (pessimism for small p , and optimism for large p) for $\gamma > 1$.

The final probability weighting function is a general functional form proposed by Prelec [1998] that exhibits considerable flexibility, nesting the first two specifications (10) and (11). This function is

$$\omega(p) = \exp\{-\eta(-\ln p)^\varphi\}, \tag{12}$$

and is defined for $0 < p \leq 1$, $\eta > 0$ and $0 < \varphi < 1$. When $\varphi = 1$ this function collapses to the Power function $\omega(p) = p^\eta$. Andersen, Harrison, Lau and Rutström [2011] point out that this function is not well-behaved numerically, in the sense of having reliable numerical derivatives, when $\varphi \rightarrow 1$.⁶ Indeed, when $\varphi = 1$ this function collapses to the Power function

$$w(p) = p^\eta \tag{10'}$$

There are several solutions to this problem of numerical instability, and the simplest is to estimate a generalization that nests (12) inside a power function with parameter γ :

$$w(p) = [\exp\{-\eta(-\ln p)^\varphi\}]^\gamma, \tag{13}$$

so that we get the numerical stability of (10) when (12) collapses because $\varphi \rightarrow 1$, and yet we do not rule out the flexibility of (12) when appropriate.⁷ We call this the Power-Prelec probability weighting function. We reference the Prelec function in the main text, because the Power-Prelec is just an extension to achieve the same specification in a more robust numerical manner.

⁶ The numerical problems are easy to see. To ensure that $0 < \varphi < 1$, we estimate the parameter φ and then define φ by $1/(1+\exp(\varphi))$ within the maximum likelihood evaluator. Hence the algorithm searching for maximum likelihood parameter values can vary φ between $\pm\infty$ and ensure that the constraint on φ is always satisfied. However, if the maximum likelihood value of φ is 1, then arbitrarily large negative values of φ will generate numerical values of φ close to 1. Thus large swings in φ can occur with the same likelihood value, disrupting the gradient methods used to locate a maximum.

⁷ The other numerical solution is to generate “profile likelihoods” in which φ is set parametrically to 1, 0.99, 0.98... 0.9, and evaluate the likelihood conditional on that assumed value and the maximum likelihood values of other parameters. Inspection of these likelihoods can then verify that the maximum likelihood value of φ is indeed 1.

We explain in the text that the Power function has a better likelihood than the Inverse-S function, and that the Prelec function essentially validates the use of the Power. Figure B1 in Appendix B shows that the estimate of γ for the Power function is 0.71 when applied solely to the risk choices over objective probabilities, with a log-likelihood value of -3927. The comparable estimate of γ for the Inverse-S function is 0.95, with a worse log-likelihood value of -3940. These qualitative results persist when one jointly estimates objective and subjective probabilities. Figure B3 considers the flexible Prelec specification⁸ when we undertake that joint estimation, showing that the preferred specification is clearly “power-like.”

The same behavioral noise specification is used as in the EUT model, replacing EU_i with RDU_i , leading to the definition of a comparable latent index ∇RDU . The likelihood specification for the RDU model is therefore

$$\ln L(\hat{r}, \hat{\alpha}, \hat{\gamma}, \hat{\mu}; y, \mathbf{X}) = \sum_i [(\ln(\nabla RDU) \times \mathbf{I}(y_i=1)) + (\ln(1 - \nabla RDU) \times \mathbf{I}(y_i = -1))] \quad (11)$$

and entails the estimate of \hat{r} , $\hat{\alpha}$, $\hat{\gamma}$ and $\hat{\mu}$ using maximum likelihood. Individual heterogeneity is allowed for by estimating the parameters \hat{r} , $\hat{\alpha}$ and $\hat{\gamma}$ as linear functions of the demographic characteristics defined earlier.

Additional References

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⁸ For reasons just explained, we used the numerically robust Power-Prelec function in all estimations, but refer to it as the Prelec function.

Appendix D: Lottery Parameters

The parameters for the 45 binary choice lotteries are listed below. The first 4 columns are the 4 monetary prizes offered. The next 4 columns are the probabilities for the corresponding outcome in the left lottery presented to subjects, and the final 4 columns are the probabilities for the corresponding outcome in the right lottery presented to subjects. One can see that the first 15 lotteries listed here have low stakes, the second 15 have medium stakes, and the last 15 have high stakes.

prize0	prize1	prize2	prize3	p0left	p1left	p2left	p3left	p0right	p1right	p2right	p3right
0	.33	.67	1	0	.26	.37	.37	0	.38	0	.62
0	.33	.67	1	0	.5	.5	0	.13	.25	.62	0
0	.33	.67	1	0	.62	.38	0	.13	.25	.62	0
0	.33	.67	1	.13	0	.25	.62	0	0	.75	.25
0	.33	.67	1	.13	.25	.62	0	0	.5	.5	0
0	.33	.67	1	.13	.25	.62	0	0	.62	.38	0
0	.33	.67	1	.13	.62	.25	0	.25	.25	.5	0
0	.33	.67	1	.13	.74	.13	0	.25	.5	.25	0
0	.33	.67	1	.13	.87	0	0	.37	.37	0	.26
0	.33	.67	1	.25	.75	0	0	.49	.13	0	.38
0	.33	.67	1	.38	.13	.49	0	.13	.74	.13	0
0	.33	.67	1	.38	.62	0	0	.75	0	0	.25
0	.33	.67	1	.5	.25	.25	0	.38	.62	0	0
0	.33	.67	1	.62	0	.38	0	.75	0	0	.25
0	.33	.67	1	.75	0	0	.25	.38	.62	0	0
0	3.33	6.67	10	0	0	.75	.25	.13	0	.25	.62
0	3.33	6.67	10	0	0	1	0	0	.13	.49	.38
0	3.33	6.67	10	0	0	1	0	.13	0	.49	.38
0	3.33	6.67	10	0	.13	.49	.38	0	0	1	0
0	3.33	6.67	10	0	.25	.75	0	0	.5	0	.5
0	3.33	6.67	10	0	.38	0	.62	0	.26	.37	.37
0	3.33	6.67	10	.25	.25	.5	0	.13	.62	.25	0
0	3.33	6.67	10	.25	.5	.25	0	.13	.74	.13	0
0	3.33	6.67	10	.37	.37	0	.26	.13	.87	0	0
0	3.33	6.67	10	.38	0	.13	.49	.25	0	.75	0
0	3.33	6.67	10	.38	0	.62	0	.26	.37	.37	0
0	3.33	6.67	10	.38	.62	0	0	.5	.25	.25	0
0	3.33	6.67	10	.38	.62	0	0	.62	.25	0	.13
0	3.33	6.67	10	.62	0	0	.38	.38	.62	0	0
0	3.33	6.67	10	.75	0	0	.25	.62	0	.38	0
0	33.33	66.67	100	0	0	1	0	0	.13	.38	.49
0	33.33	66.67	100	0	0	1	0	.13	0	.13	.74
0	33.33	66.67	100	0	.13	.38	.49	0	0	1	0
0	33.33	66.67	100	0	.38	.62	0	0	.62	0	.38
0	33.33	66.67	100	0	.5	0	.5	0	.25	.75	0
0	33.33	66.67	100	0	.62	0	.38	0	.38	.62	0
0	33.33	66.67	100	0	1	0	0	.13	.49	.38	0
0	33.33	66.67	100	.13	0	.13	.74	0	0	1	0
0	33.33	66.67	100	.13	0	.49	.38	0	0	1	0
0	33.33	66.67	100	.13	.49	.38	0	0	1	0	0
0	33.33	66.67	100	.13	.74	.13	0	.38	.13	.49	0
0	33.33	66.67	100	.25	0	.75	0	.38	0	.13	.49
0	33.33	66.67	100	.26	.37	.37	0	.38	0	.62	0
0	33.33	66.67	100	.38	.62	0	0	.62	0	0	.38
0	33.33	66.67	100	.49	.13	0	.38	.25	.75	0	0