Appendix to

Temporal patterns of populations in a warming world: a modelling framework

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In the following, parameter values are listed for each application separately. In application 3 and 5, numerous parameters adopted dependencies on model-specific states. Hence, these relationships are also furnished and the meaning of introduced parameters can be found in the parameter list.

App. 1: Growth and mortality at varying temperatures

Parameter	**Value**
$Q_{10\ mort}$	1.06 °C^{-1}
$Q_{10\ growth}$	1.5 °C^{-1}
r_{max}	0.024 d^{-1}
T_{max}	40 °C
T_{min}	0 °C
$T_{opt\ mort}$	6.5 °C
$T_{opt\ growth}$	17 °C
μ	0.003 d^{-1}

App. 2: Seasonal regulation mechanisms

Parameter	**Value**
$Q_{10\ 52°\ N}$	2.1 °C^{-1}
$Q_{10\ 60°N}$	1.9 °C^{-1}
$R_{max\ 52°N}$	0.036 mm d^{-1}
$R_{max\ 60°N}$	0.028 mm d^{-1}
$T_{opt\ 52°N}$	28.7 °C
$T_{opt\ 60°N}$	26.5 °C
$D_{crit\ 52°N}$	12.5 h
$D_{crit\ 60°N}$	16.0 h
T_{max}	40 °C

App. 3: Dispersal induced by extreme drought events

Growth equation:

$$\frac{ds}{dt} = \gamma\,(s_m - s)\,\Phi_T(T) = g(s,T) \tag{1}$$

Distribution of newborn:

$$\Pi(s) = \frac{1}{\sqrt{2\,\pi\,s_d{}^2}}\,e^{\frac{-(s-s_n)^2}{2\,s_d{}^2}} \tag{2}$$

Size-dependent mortality:

$$m_s(s) = \mu_s\,e^{-\left(\frac{s}{s_{c1}}\right)^{\beta}} + 1 - e^{-\left(\frac{s}{s_{c2}}\right)^{\beta}} \tag{3}$$

Temperature-dependent mortality:

$$m_T(T) = \alpha_1\,e^{\alpha_2\,T} \tag{4}$$

Water depth dependent mortality:

$$m_W(W) = \mu_W\,e^{\left(\frac{-W}{\tau_1}\right)^4} \tag{5}$$

Parameter	**Value**	**Meaning**
$Q10_{growth}$	1.6 °C^{-1}	shape parameter of temperature response, growth
$T_{max,growth}$	26 °C	maximum temperature, growth
$T_{opt\ growth}$	18 °C	optimum temperature, growth
$Q10_{birth}$	2 °C^{-1}	shape parameter of temperature response, reproduction
$T_{max,\ birth}$	26 °C	maximum temperature, reproduction
$T_{opt,\ birth}$	18 °C	optimum temperature, reproduction
μ_W	0.08	water depth dependent mortality rate
μ_s	0.044 d^{-1}	size dependent mortality rate
b(s,n)	0.0045 d^{-1}	total number of offspring
s_{min}	3 mm	minimum size for reproduction
s_{max}	12 mm	maximum size for reproduction
b_{max}	0.0016 mm^{-2} d^{-1}	maximum birth rate
κ	15	shape parameter for water depth response
τ_1	10 cm	critical water depth (taking refuge)
τ_2	30 cm	critical water depth (resumption)
τ	40 d	time lag water depth response
k_{um}	0.03 d^{-1}	dispersal rate between population in

		upstream and main reach
k_{mu}	0.01 d^{-1}	dispersal rate between population in main and upstream reach
k_{mr}	0.10 d^{-1}	dispersal rate between population in main reach and refuge
k_{rm}	0.10 d^{-1}	dispersal rate between population in refuge and main reach
k_{md}	0.03 d^{-1}	dispersal rate between population in main and downstream reach
k_{dm}	0.05 d^{-1}	dispersal rate between population in downstream and main reach
γ	0.0089 d^{-1}	growth rate
s_m	12 mm	maximal body length
D	0.0023 $mm^2\ d^{-1}$	Diffusion coefficient
α_1	0.0041 d^{-1}	empirical Parameter for the temperature dependence of mortality
α_2	0.0698 $°C^{-1}$	empirical Parameter for the temperature dependence of mortality
s_{c1}	2 mm	critical size juvenile mortality
s_{c2}	12 mm	critical size adult mortality
κ_s	25	shape parameter size-dependent mortality
s_n	0.7 mm	mean size at birth
s_d	0.2 mm	standard deviation of size at birth

App. 4: Biotic interactions

Parameter	**Value**
$Q_{10\ 52°\ N}$	2.1 $°C^{-1}$
$Q_{10\ 60°N}$	1.9 $°C^{-1}$
$R_{max\ 52°N}$	0.036 mm d^{-1}
$R_{max\ 60°N}$	0.028 mm d^{-1}
$T_{opt\ 52°N}$	28.7 °C
$T_{opt\ 60°N}$	26.5 °C
$D_{crit\ 52°N}$	12.5 h
$D_{crit\ 60°N}$	16.0 h
T_{max}	40 °C
$a(s)$ = const = a	1
K	45

App. 5: Coupled consumer-resource dynamics

The consumer-resource dynamics is described by a coupled system of three balance equations:

$$\frac{\partial n_{\mathrm{G}}}{\partial t} = -\frac{\partial g\, u_{\mathrm{G}}}{\partial s} - m\, n_{\mathrm{G}} + B\, \Pi_{\mathrm{s}}(s) \qquad (1)$$

$$\frac{\partial n_{\mathrm{L}}}{\partial t} = -\frac{\partial c\, n_{\mathrm{L}}}{\partial q} - (f_{\mathrm{Shr}} + a)\; n_{\mathrm{L}} + J\, \Pi_{\mathrm{q}}(q) \qquad (2)$$

$$\frac{dA}{dt} = -\eta\, B - f_{\mathrm{Rea}}\, A + \epsilon \int f_{\mathrm{Shr}} n_{\mathrm{L}} dq \qquad (3)$$

Growth of G. pulex population:

$$g = \min\left(0,\, \gamma\, (s_{\max}\, \rho_g - s)\, \phi_{\mathrm{g}}(T)\right) \qquad (4)$$

Total mortality:

$$m = 1 - (1-\mu)\,(1 - m_{\mathrm{T}}(T))\,(1 - m_{\mathrm{s}}(s))\,(1 - m_{\mathrm{L}}(t)) \qquad (5)$$

Temperature-dependent mortality:

$$m_T(T) = \frac{\mu}{1-\mu}\,(e^{\alpha T} - 1) \qquad (6)$$

Size-dependent mortality:

$$m_s(s) = \mu_j\, e^{-\left(\frac{s}{s_{c1}}\right)^{\beta_1}} + 1 - e^{-\left(\frac{s}{s_{c2}}\right)^{\beta_2}} \qquad (7)$$

Food-dependent mortality:

$$m_{\mathrm{L}}(t) = 1 - e^{-\left(\frac{M(t)}{\tau_{\mathrm{s}}}\right)^{\beta_{\mathrm{L}}}} \tag{8}$$

Reproduction:

$$B = \phi_{\mathrm{b}}(T)\, \rho_{\mathrm{b}} \int_{s_{\mathrm{b}}}^{s_{\max}} b_{\max}\, s^2\, u_{\mathrm{G}}(t,x)\, dx \tag{9}$$

Size-specific reproduction rate:

$$b_{\max} = \frac{1}{2} \frac{b}{365}\, r_{\max} \tag{10}$$

Conditioning of leaf litter:

$$c = c_{\max} \phi_{\mathrm{c}}(T)\, (1 - H(q - 0.9)) \tag{11}$$

with the Heaviside function H switching off further conditioning beyond a quality $q = 0.9$. Alternative decomposition is modelled as

Net leaf fall:

$$J = F(t)(1 - l_c) \tag{12}$$

Alternative decomposition:

$$a = a_{\max}\, \phi_{\mathrm{c}}(T) \tag{13}$$

Loss of leaf litter through shredding:

$$f_{\mathrm{Shr}} = \frac{e_{\mathrm{c}}\, \phi_{\mathrm{g}}(T)\, P_{\mathrm{DW}}}{1 + \int_0^1 \frac{e_{\mathrm{c}}}{I_{\max}(q)}\, u_{\mathrm{L}}(t,q)\, dq} \tag{14}$$

Maximum feeding rate:

$$I_{\max}(q) = (I_{\mathrm{ceil}} - I_{\mathrm{base}})\, q + I_{\mathrm{base}} \tag{15}$$

Reallocation of energy:

$$f_{\mathrm{Rea}} = \begin{cases} \frac{e_{\mathrm{c}}\, \phi_{\mathrm{g}}(T)\, P_{\mathrm{DW}}}{1 + \frac{e_{\mathrm{c}}}{I_{\mathrm{ceil}}}\, A} & \text{if } \; L(t) < L_{\mathrm{c}} \\ 0 & \text{else} \end{cases} \tag{16}$$

Energy response of reproduction:

$$\rho_b = \frac{A}{A + \eta_E\, \int_{s_b}^{s_{\max}} b_{\max}\, s^2\, u_{\mathrm{G}}(t,x)\; ds} \tag{17}$$

Energy response of growth:

$$\rho_g = \begin{cases} \frac{Q(t)+A(t)}{Q(t)+A(t)+\frac{I_{\mathrm{ceil}}}{e_{\mathrm{c}}}} & \text{if } \; L(t) < L_{\mathrm{c}} \\ \frac{Q(t)}{Q(t)+\frac{I_{\mathrm{ceil}}}{e_{\mathrm{c}}}} & \text{else} \end{cases} \tag{18}$$

Total leaf litter content:

$$L(t) = \int_0^1 n_{\mathrm{L}}(t,q)\, dq \tag{19}$$

Total quality weighted leaf litter content:

$$Q(t) = \int_0^1 q\, n_{\mathrm{L}}(t,q)\, dq \tag{20}$$

Total abundance of G. pulex:

$$N(t) = \int_0^{s_{\max}} n_{\mathrm{G}}(t,s)\, ds \tag{21}$$

Total shredding mass of G. pulex:

$$P_{\mathrm{DW}}(t) = \int_{s_{\mathrm{f}}}^{s_{\max}} w_{\mathrm{D}}(s)\, n_{\mathrm{G}}(t,s)\, ds \tag{22}$$

Size-dependent dry weight of G. pulex:

$$w_{\mathrm{D}}(s) \;=\; 0.0024\, s^3 \tag{23}$$

$F(t)$ is leaf litter fall. $\Pi_{\mathrm{s}}(s)$ and $\Pi_{\mathrm{q}}(q)$ introduce the newborns and recently fallen leaves with a normal distribution.

The starvation mortality depends on the state of a timer, $M(t)$, which counts consecutive days without food (i.e. days with $L(t) < L_{\mathrm{c}}$).

Symbol	**Unit**	**Meaning**
a_{max}	0.1 d^{-1}	Microbial decay rate
b	10 y^{-1}	Number of broods per year
e_c	0.0001 m^2 d^{-1} mg^{-1}	Encounter rate
g_{max}	0.19 d^{-1}	Maximum rate of leaf conditioning
I_{base}	0.0001 g mg^{-1} d^{-1}	Feeding rate on unconditioned leaves
I_{ceil}	0.0004 g mg^{-1} d^{-1}	Feeding rate on maximally conditioned leaves
L_c	0.1 g m^{-2}	Critical total leaf litter for starvation
l_c	0.1	Fraction of leached leaf fall
q_{10b}	4.4 °C^{-1}	Temperature coefficient reproduction

q_{10f}	1,85 °C^{-1}	Temperature coefficient feeding and growth
q_{10L}	2 °C^{-1}	Temperature coefficient decay
r_{max}	0.1985 mm^{-2}	Maximum reproduction rate
s_b	6 mm	Minimum size for reproduction
s_{c1}	3 mm	Critical size of the juvenile mortality
s_{c2}	18 mm	Critical size of the senile mortality
s_f	3 mm	Minimum size for litter feeding
s_{max}	18.89 mm	Maximum size
T_{maxb}	29 °C	Maximum temperature reproduction
T_{maxf}	40 °C	Maximum temperature feeding and growth
T_{maxL}	40 °C	Maximum temperature decay
T_{optb}	14.4 °C	Optimal temperature reproduction
T_{optf}	17.44 °C	Optimal temperature feeding and growth
T_{optL}	30 °C	Optimal temperature decay
α	0.155 °C^{-1}	Shape parameter mortality
β_1	25	Shape parameter mortality
β_2	10	Shape parameter mortality
β_L	8	Shape parameter mortality
γ	0.0062 d^{-1}	Bertalanffy growth rate
ε	0.7	shredding efficiency
η	0.006 g	Energy loss per newborn
η_E	0.006 g d	Energy loss per newborn; rate response function
μ	0.0018 d^{-1}	Basic mortality
μ_j	0.002 d^{-1}	Juvenile mortality
τ_s	50 d	Average survival time under starvation